Revealed Preferences over Risk and Uncertainty

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May 11, 2015

Revealed Preference Analysis

Let $\mathcal{O} = \{(p^t, x^t)\}_{t \in \mathcal{T}}$ be a set of observations drawn from a consumer. Each observation consists of a price vector $p^t = (p_1^t, p_2^t, \ldots, p_\ell^t) \gg 0$

and a consumption bundle $x^t = (x_1^t, x_2^t, \ldots, x_\ell^t) \geq 0$.

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Now suppose that we have access to $\mathcal{O} = \{(p^t, x^t)\}_{t \in \mathcal{T}}$, where p^t is a vector of state prices and x^t is a contingent consumption bundle.

Aim of this paper: to develop and implement revealed preference tests on $\mathcal{O}$ for different models of choice under risk and uncertainty.

GARP

Given the data set $\mathcal{O} = \{(p^t, x^t)\}_{t\in\mathcal{T}}$, we denote the set of observed consumption bundles or demands by $\mathcal{D}$, i.e., $\mathcal{D} = \{x^t\}_{t\in\mathcal{T}}$.

For any x^t, $x^s \in \mathcal{D}$, we say that x^t is directly revealed preferred to x^s if $p^t \cdot x^s \leq p^t \cdot x^t$. [Notation: $x^t \succeq^* x^s$.]

If $p^t \cdot x^s < p^t \cdot x^t$, we say that x^t is directly revealed strictly preferred to x^s. [Notation: $x^t \succ^* x^s$.]

Motivation: For an agent maximizing a nonsatiated utility function U,

$$x^t \succeq^* x^s \implies U(x^t) \geq U(x^s) \text{ and}$$

$$x^t \succ^* x^s \implies U(x^t) > U(x^s).$$

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Definition: A data set $\mathcal{O} = \{(p^t, x^t)\}_{t \in \mathcal{T}}$ obeys the Generalized Axiom of Revealed Preference (GARP) if whenever there is a sequence of observations (p^{t_i}, x^{t_i}) (for $i = 1, 2, \ldots, n$) satisfying

$$x^{t_1} \succeq^* x^{t_2},\ x^{t_2} \succeq^* x^{t_3},\ \ldots,\ x^{t_{n-1}} \succeq^* x^{t_n},\ x^{t_n} \succeq^* x^{t_1},$$

then $\succeq^*$ cannot be replaced with $\succ^*$ anywhere in the chain.

The necessity of GARP

Lemma: Any data set $\mathcal{O} = \{(p^t, x^t)\}_{t \in \mathcal{T}}$ collected from an agent who is maximizing a locally nonsatiated utility function must obey GARP.

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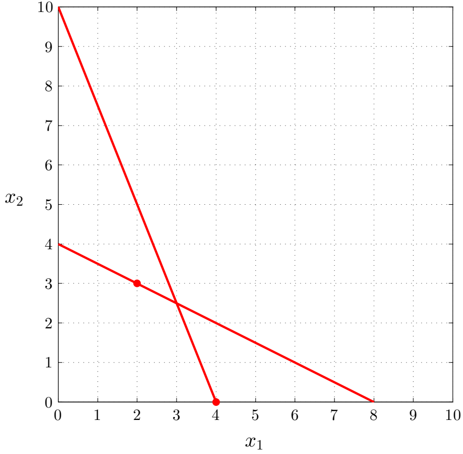

Violation of GARP: $x^1 \succ^* x^2$ and $x^2 \succ^* x^1$.

GARP and Afriat's Theorem

Lemma: Whenever a data set $\mathcal{O} = \{(p^t, x^t)\}_{t \in \mathcal{T}}$ is collected from an agent who is maximizing a nonsatiated preference, it obeys GARP.

Afriat's Theorem: Suppose that $\mathcal{O} = \{(p^t, x^t)\}_{t \in \mathcal{T}}$ satisfies GARP. Then there are real numbers ϕ^t and $\lambda^t \; > 0$ (for every $t \in \mathcal{T}$) that solve the following system of linear inequalities:

$$\phi^t \leq \phi^k + \lambda^k p^k \cdot (x^t - x^k) \text{ for all } k \neq t.$$

Furthermore, $\mathcal{O}$ can be rationalized by $U : \mathbb{R}^\ell_+ \to \mathbb{R}$ taking the form

$$U(x) = \min_{t \in \mathcal{T}} \{\phi^t + \lambda^t p^t \cdot (x - x^t)\}.$$

Two things to notice about this result:

(1) Solving linear inequalities is computationally straightforward.

(2) U is increasing, concave, and continuous.

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(3) $\mathcal{O}$ is rationalizable by a utility function U that is continuous, strictly increasing, and concave.

Notice the difference between (1) and (3): continuity, strict monotonicity, and concavity are obtained 'for free'.

Contingent consumption and rationalizability

Now suppose that an agent is choosing contingent consumption, so

$$p^t = (p_1^t, p_2^t, \ldots, p_S^t)$$

is a vector of state prices, with $\mathcal{S} = \{1, 2, \ldots, S\}$ the set of states, and

$$x^t = (x_1^t, x_2^t, \ldots, x_S^t)$$

is a bundle of contingent consumption.

Suppose we know the probability of state s is $\pi_s > 0$ (for all $s \in \mathcal{S}$). How do we test for rationalizability by expected utility, i.e., there is an increasing and continuous utility function $u : \mathbb{R}_+ \to \mathbb{R}$ such that

$$\sum_{s \in \mathcal{S}} \pi_s u(x_s^t) \geq \sum_{s \in \mathcal{S}} \pi_s u(x_s)$$

for all $x \in \mathcal{B}^t = \{x \in \mathbb{R}_+^S : p^t \cdot x \leq p^t \cdot x^t\}$, with $t \in \mathcal{T}$?

Rationalizability by Expected Utility

The standard approach of Varian (1983) and Green and Srivastava (1986) is to assume that u is increasing, concave, and continuous.

Optimality implies that there is some $\lambda^t > 0$ (for all $t \in \mathcal{T}$) such that

$$\pi_s u'(x_s^t) = \lambda^t p_s^t \text{ for all } (s,t) \in \mathcal{S} \times \mathcal{T}.$$

Therefore, for each $(s,t) \in \mathcal{S} \times \mathcal{T}$, there is some $\beta_s^t > 0$ such that

$$\frac{\pi_1 \beta_1^t}{p_1^t} = \frac{\pi_2 \beta_2^t}{p_2^t} = \cdots = \frac{\pi_S \beta_S^t}{p_S^t} \text{ for all } t \in \mathcal{T}.$$

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Theorem: The data set $\mathcal{O} = \{(p^t, x^t)\}_{t \in \mathcal{T}}$ is EU-rationalizable with $\pi = (\pi_s)_{s \in \mathcal{S}}$ by an increasing, concave, and continuous utility function u if and only if there is some $\beta_s^t > 0$ (for all $(s,t) \in \mathcal{S} \times \mathcal{T}$) such that

(1) whenever $x_s^t > x_{s'}^{t'}$, then $\beta_s^t \leq \beta_{s'}^{t'}$,

(2) for every $t \in \mathcal{T}$, $\pi_s \beta_s^t / p_s^t = \pi_{s'} \beta_{s'}^t / p_{s'}^t$.

Rationalizability by Expected Utility

This approach gives a simple test for EU-rationalizability:

all that one needs to do is find β_s^t (for all $(s, t) \in \mathcal{S} \times \mathcal{T}$) that solve the linear inequalities (1) and (2).

But this test relies on the *sufficiency* of the first order condition and this only holds when the preference over $\mathbb{R}_+^S$ is convex and the budget set is convex.

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- Convexity of the preference excludes risk loving behavior.
- Convexity of the budget set
  - excludes nonlinear pricing,
  - makes it difficult to extend the test in order to measure the 'size' of departures from EU-rationality, which is potentially limiting in many empirical applications.

Our Approach to Testing EU-Rationalizability

Definition: The data set $\mathcal{O} = \{(p^t, x^t)\}_{t \in \mathcal{T}}$ is EU-rationalizable with $\pi = (\pi_s)_{s \in \mathcal{S}}$ if there is an increasing and continuous utility function $u : \mathbb{R}_+ \to \mathbb{R}$ such that, at every observation $t \in \mathcal{T}$,

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(3) It is applicable even when budget sets are nonconvex.

(4) It can be adapted to measure the size/significance of departures from a particular model.

The Lattice Test of EU-Rationalizability

Suppose that $\pi = (1/2, 1/2)$ and we have the following observations:

$$x^1 = (2,\ 5) \text{ at } p^1 = (5, 2),$$
$$x^2 = (6,\ 1) \text{ at } p^2 = (1, 2),$$
$$x^3 = (4,\ 3) \text{ at } p^3 = (4, 3).$$

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In this case, $\mathcal{X} = \{0, 1, 2, 3, 4, 5, 6\}$.

Construct the finite lattice $\mathcal{L} = \mathcal{X}^2 \subset \mathbb{R}_+^2$.

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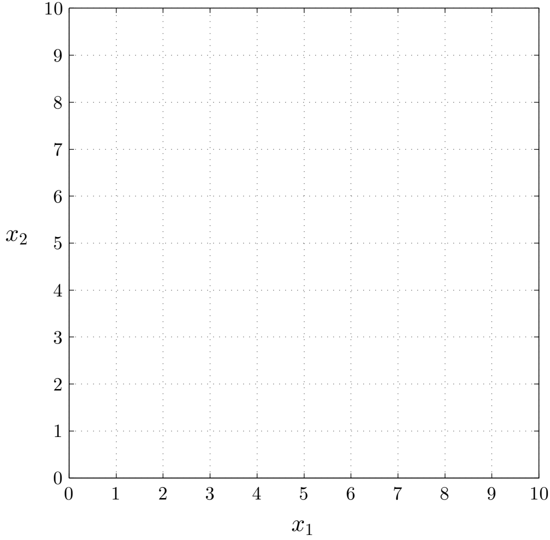

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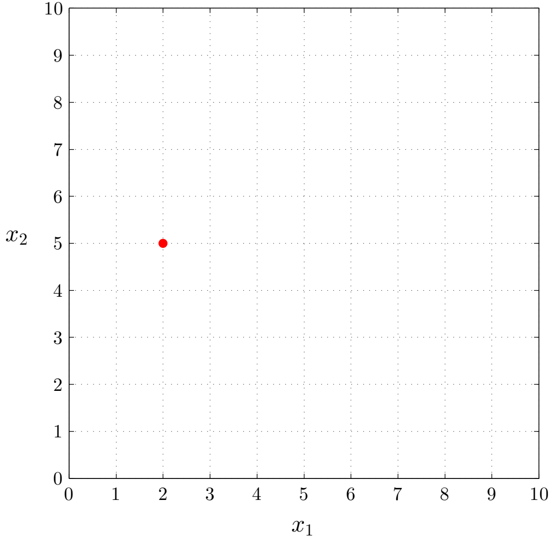

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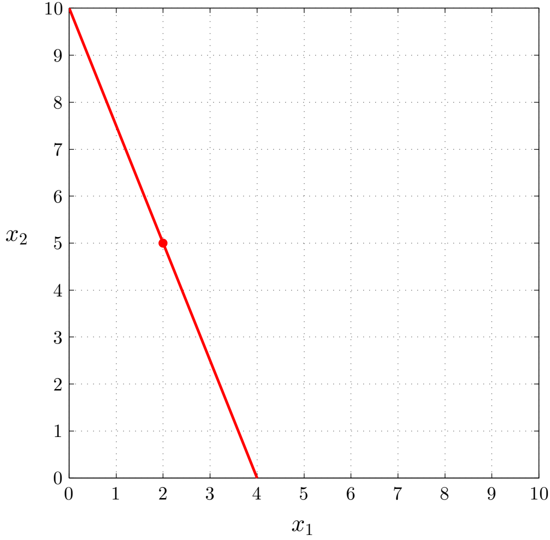

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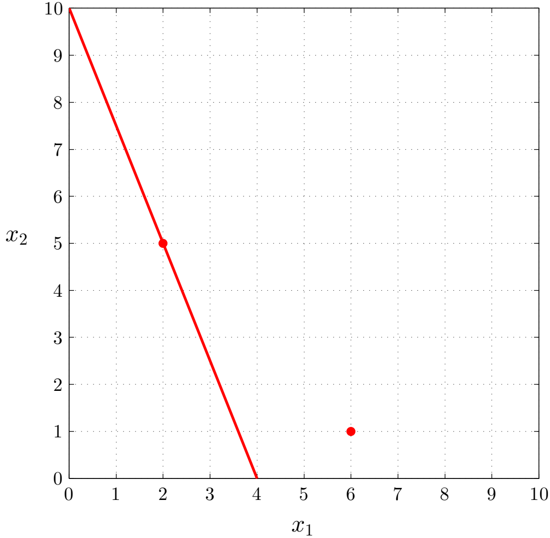

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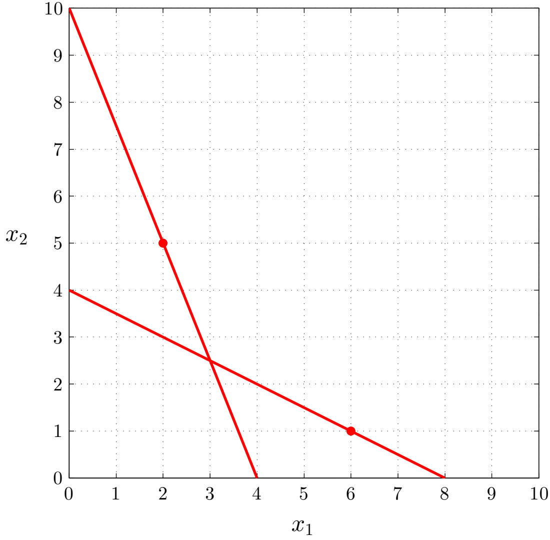

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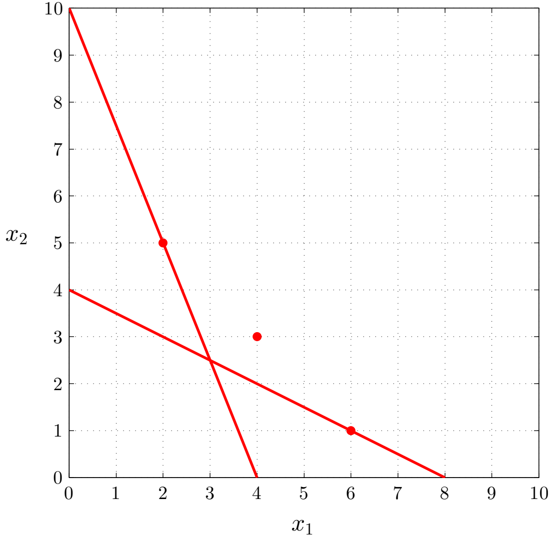

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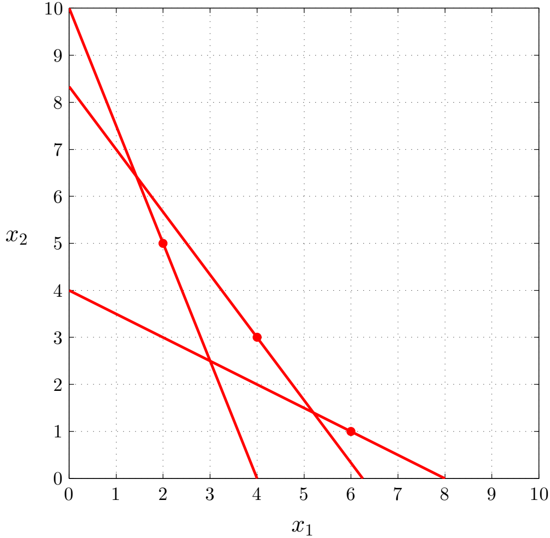

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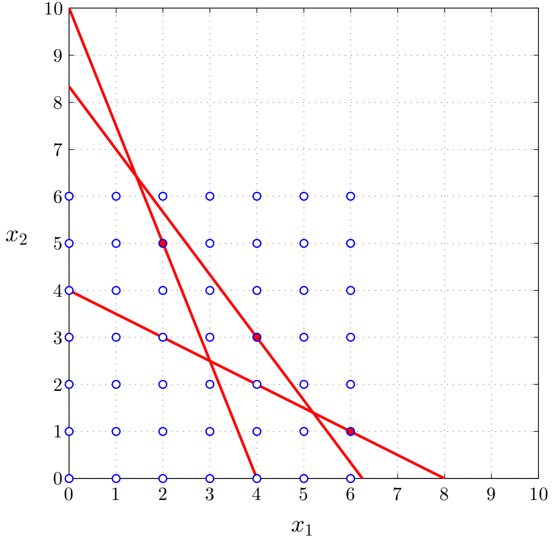

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Define $\mathcal{X} = \{x_s^t : (s, t) \in \mathcal{S} \times \mathcal{T}\} \cup \{0\}$. Here, $\mathcal{X} = \{0, 1, 2, 3, 4, 5, 6\}$.

Construct the finite lattice $\mathcal{L} = \mathcal{X}^2$.

For EU-rationalizability, it is clearly necessary that there are real numbers $\bar{u}(0) < \bar{u}(1) < \cdots < \bar{u}(6)$, such that, at every $t \in \{1, 2, 3\}$,

$$\frac{1}{2}\bar{u}(x_1^t) + \frac{1}{2}\bar{u}(x_2^t) \geq \frac{1}{2}\bar{u}(x_1) + \frac{1}{2}\bar{u}(x_2) \text{ for all } x \in \mathcal{B}^t \cap \mathcal{L},$$

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It is also sufficient to guarantee EU-rationalizability by an increasing and continous function $u : \mathbb{R}_+ \to \mathbb{R}$ that extends $\bar{u} : \mathcal{X} \to \mathbb{R}$.

So we only need to check EU-rationalizability on a finite lattice, which is a straightforward linear test.

The Lattice Test of EU-Rationalizability

Theorem: The data set $\mathcal{O} = \{(p^t, x^t)\}_{t\in\mathcal{T}}$ is EU-rationalizable with $\pi = (\pi_s)_{s\in\mathcal{S}}$ if there is an increasing utility function $\bar{u} : \mathcal{X} \to \mathbb{R}$ such that, at every observation $t \in \mathcal{T}$,

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$$\sum_{s\in\mathcal{S}} \pi_s \bar{u}(x_s^t) > \sum_{s\in\mathcal{S}} \pi_s \bar{u}(x_s) \text{ for all } x \in (\mathcal{B}^t \backslash \partial\mathcal{B}^t) \cap \mathcal{L}.$$

Intuition: First we replace $\bar{u}$ with the step function $\hat{u} : \mathbb{R}_+ \to \mathbb{R}$ so that $\hat{u}(y) = \bar{u}(y)$ for all $y \in \mathcal{X}$ and $\hat{u}$ is constant between values of $\mathcal{X}$. Clearly, $\hat{u}$ rationalizes the data in the sense that

$$\sum_{s\in\mathcal{S}} \pi_s \hat{u}(x_s^t) \geq \sum_{s\in\mathcal{S}} \pi_s \hat{u}(x_s) \text{ for all } x \in \mathcal{B}^t.$$

The only problem is that $\hat{u}$ is neither increasing nor continuous. But it is possible to find another utility function u, arbitrarily close to $\hat{u}$, that is increasing and continuous which also rationalizes the data.

The Lattice Test of EU-Rationalizability

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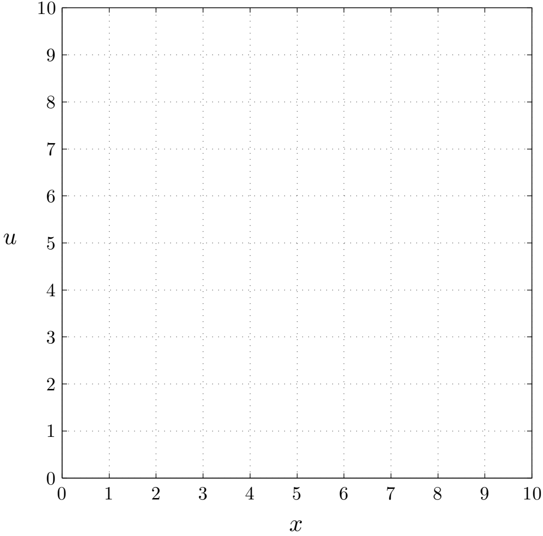

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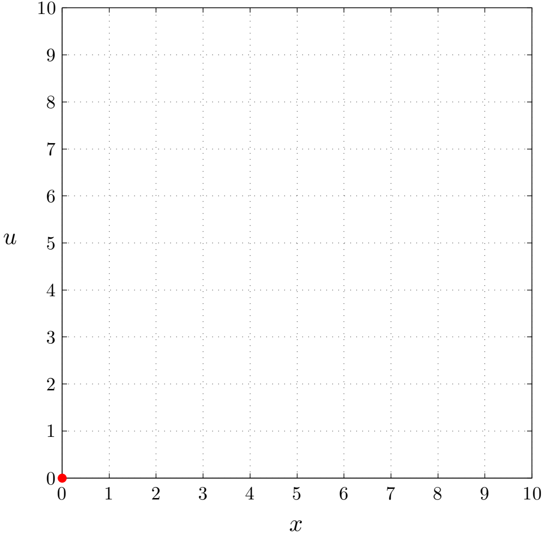

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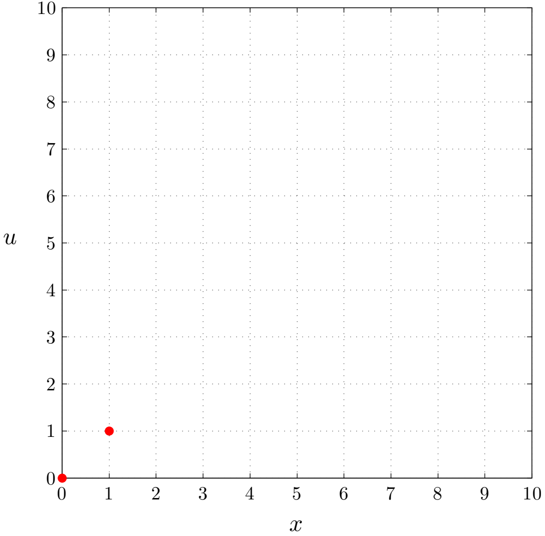

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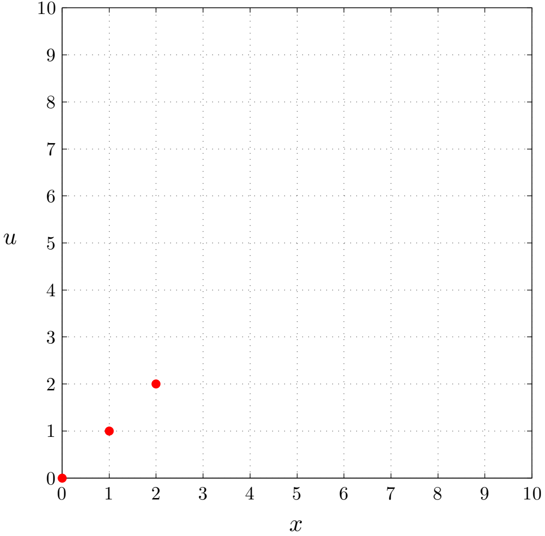

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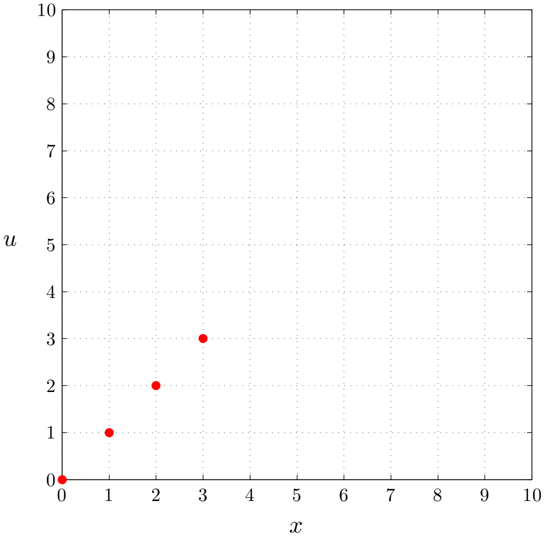

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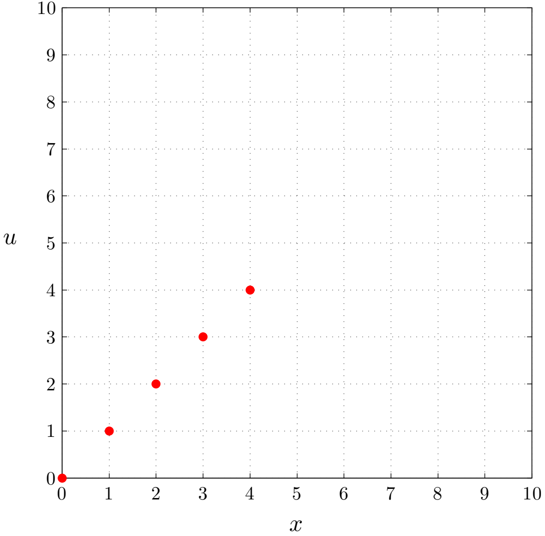

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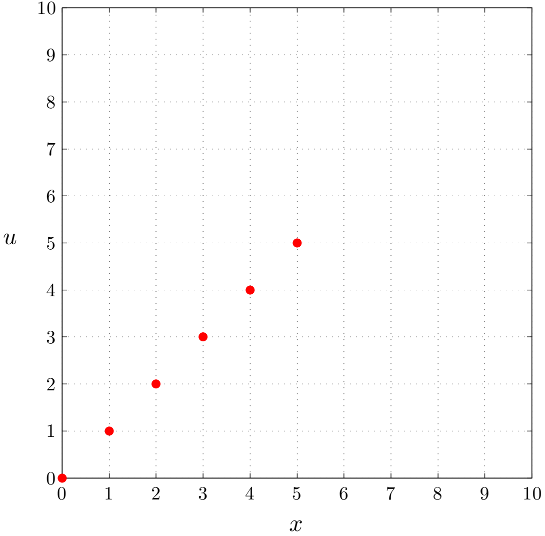

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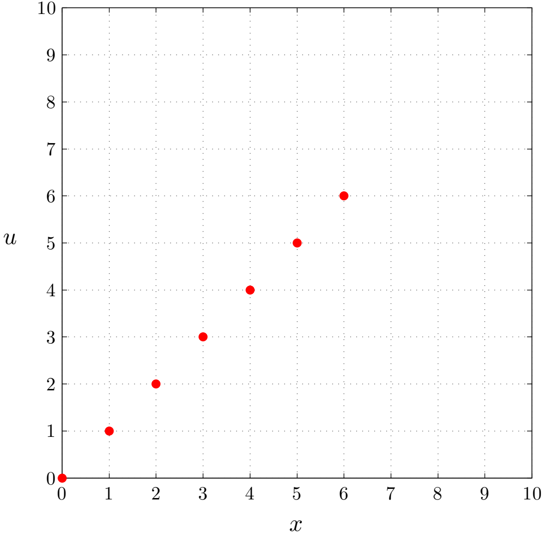

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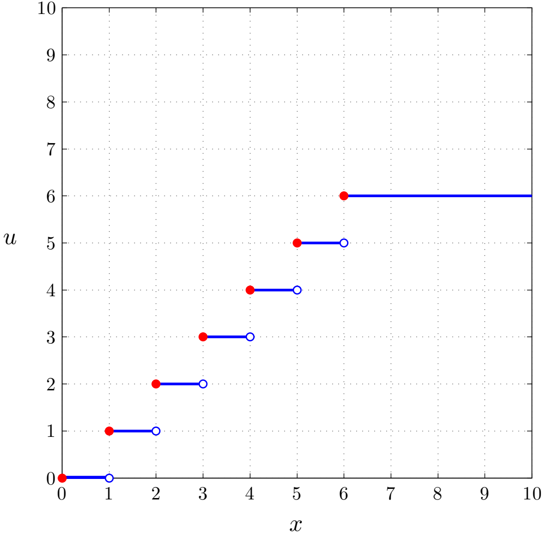

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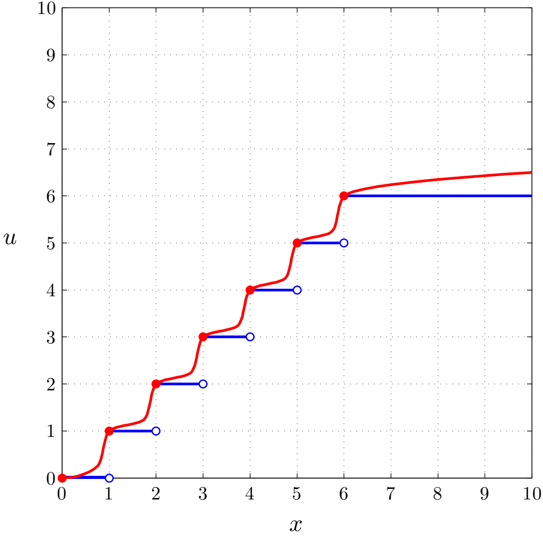

The Lattice Test in More General Settings

In many models of choice under risk/uncertainty, utility over contingent consumption bundles $x = (x_1, x_2, ..., x_S)$ takes the following form:

there is an increasing and continuous function $\phi : \mathbb{R}^S \to \mathbb{R}$, such that

$$U(x) = \phi(u(x_1), u(x_2), \ldots, u(x_S))$$

for some increasing and continuous (Bernoulli) utility function $u : \mathbb{R}_+ \to \mathbb{R}$.

Different models specify different functional forms for ϕ.

This framework captures objective and subjective expected utility, rank dependent utility, reference dependent utility (Koszegi-Rabin), maxmin expected utility, and variational preferences, etc.

The Lattice Test in More General Settings

Consider a data set $\mathcal{O} = \{(x^t, \mathcal{B}^t)\}_{t \in \mathcal{T}}$, where x^t is the bundle chosen from a compact constraint set $\mathcal{B}^t$.

Given $\mathcal{O}$, we define $\mathcal{X} = \{x_s^t : (s,t) \in \mathcal{S} \times \mathcal{T}\} \cup \{0\}$

and the finite lattice $\mathcal{L} = \mathcal{X}^S$.

Theorem: The data set $\mathcal{O} = \{(p^t, x^t)\}_{t \in \mathcal{T}}$ is ϕ-rationalizable if there is an increasing utility function $\bar{u} : \mathcal{X} \to \mathbb{R}$ such that, at every $t \in \mathcal{T}$,

$$\phi(\bar{u}(x_1^t), \bar{u}(x_2^t), \ldots, \bar{u}(x_S^t)) \geq \phi(\bar{u}(x_1), \bar{u}(x_2), \ldots, \bar{u}(x_S))$$

for all $x \in \mathcal{B}^t \cap \mathcal{L}$,

$$\phi(\bar{u}(x_1^t), \bar{u}(x_2^t), \ldots, \bar{u}(x_S^t)) > \phi(\bar{u}(x_1), \bar{u}(x_2), \ldots, \bar{u}(x_S))$$

for all $x \in (\mathcal{B}^t \backslash \partial \mathcal{B}^t) \cap \mathcal{L}$.

Testing Models of Choice under Risk

In objective EU, $\phi(u_1, u_2, \ldots, u_S) = \sum_{s\in\mathcal{S}} \pi_s u_s$.

Our test involves finding $\bar{u}(y)$ (for each $y \in \mathcal{X}$) that solve

$$\sum_{s\in\mathcal{S}} \pi_s \bar{u}(x_s^t) \geq \sum_{s\in\mathcal{S}} \pi_s \bar{u}(x_s) \text{ for all } x \in \mathcal{B}^t \cap \mathcal{L}, \text{ etc.}$$

In Koszegi and Rabin (2007),

$$\phi((u_1, u_2, \ldots, u_S), \pi) = \sum_{s\in\mathcal{S}} \pi_s u_s + \frac{1}{2}(1-\lambda) \sum_{s\in\mathcal{S}} \sum_{s'\in S} \pi_s \pi_{s'} |u_s - u_{s'}|,$$

where $\lambda \in [0, 2]$.

Our test involves finding $\bar{u}(y)$ (for each $y \in \mathcal{X}$) and λ that solve a set of bilinear inequalities.

This can be implemented straightforwardly—let λ take different values on $[0, 2]$ and solve the corresponding linear problem.

Disappointment Aversion Model (Gul, 1991)

When there are two states, this is a special case of rank dependent utility where the probability of a state is distorted according to the rank of the outcome.

If $x_H \geq x_L$ and the true probability of H is π_H, then the agent behaves *as though* this probability is

$$\gamma(\pi_H) = \frac{\pi_H}{1 + (1 - \pi_H)\beta},$$

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When $\beta > 0$ (so $\gamma(\pi_H) < \pi_H$) the agent is disappointment averse; if $\beta < 0$ the agent is elation seeking; $\beta = 0$ reduces to EU.

Implementation on Models of Choice under Risk

We implement our tests using data from the portfolio choice experiment in Choi, Fisman, Gale, and Kariv (*AER*, 2007).

93 undergraduate subjects participated in the experiment at UC Berkeley, each completing 50 decision problems under *risk*.

There were two states of the world, each occurring with a *known* probability, and two Arrow-Debreu securities, one for each state.

In each decision problem, every subject was given a budget; income was normalized to one, and state prices were chosen at random.

47 subjects received a symmetric treatment, where $\pi_1 = \pi_2 = 1/2$, and 46 received an asymmetric treatment, where $\pi_1 = 1/3\,(2/3)$.

Rationalizability Results

Choi *et al.* (2007) first implemented a GARP test, and then applied a parametric model in order to test expected utility (EU) and Gul's (1991) model of disappointment aversion (DA).

We conduct a parallel set of analyses, but we maintain a completely nonparametric approach using the tests we have developed.

Treatment	GARP	DA	EU
$\pi_1 = 1/2$	12/47 (26%)	1/47 (2%)	1/47 (2%)
$\pi_1 \neq 1/2$	4/46 (9%)	1/46 (2%)	1/46 (2%)
Total	16/93 (17%)	2/93 (2%)	2/93 (2%)

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Table 1: Pass Rates

The pass rates are low across models, which is unsurprising with 50 observations per subject.

We need a way of modifying our revealed preference tests in order to measure the extent to which a model is able to explain the data.

Critical Cost Efficiency Index

We use an approach suggested by Afriat (1972) and Varian (1990).

The data set $\mathcal{O} = \{(p^t, x^t)\}_{t \in \mathcal{T}}$ is rationalizable by some family $\mathbf{U}$ if there is a utility function $U : \mathbb{R}^S_+ \to \mathbb{R}$ belonging to $\mathbf{U}$ such that

$$U(x^t) \geq U(x) \text{ for all } x \in \mathcal{B}^t = \{x \in \mathbb{R}^S_+ : p^t \cdot x \leq p^t \cdot x^t\}.$$

If no function in $\mathbf{U}$ rationalizes $\mathcal{O}$, we can make the requirement less stringent by shrinking all budget sets in $\mathcal{O}$ by a factor $\alpha \in [0, 1)$.

We find U in $\mathbf{U}$ such that $U(x^t) \geq U(x)$ for all $x \in \mathcal{B}^t(\alpha)$, where

$$\mathcal{B}^t(\alpha) = \{x \in \mathbb{R}^S_+ : x \leq x^t\} \cup \{x \in \mathbb{R}^S_+ : p^t \cdot x \leq \alpha\, p^t \cdot x^t\}.$$

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The largest α at which a data set passes the test is known as the critical cost efficiency index (CCEI) associated with $\mathcal{O}$ and $\mathbf{U}$.

Notice that $\mathcal{B}^t(\alpha)$ is not a convex set.

Critical Cost Efficiency Index

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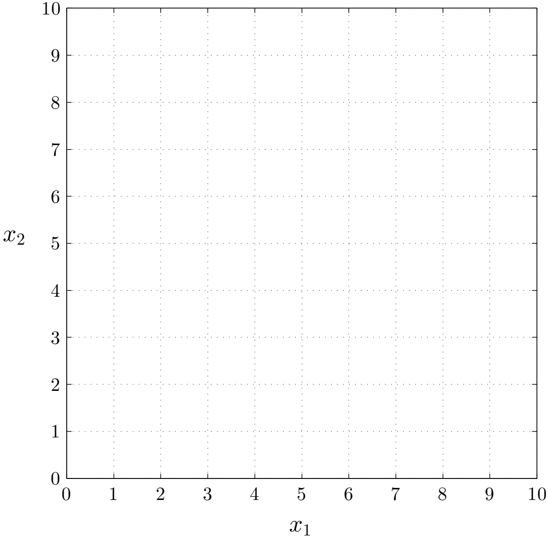

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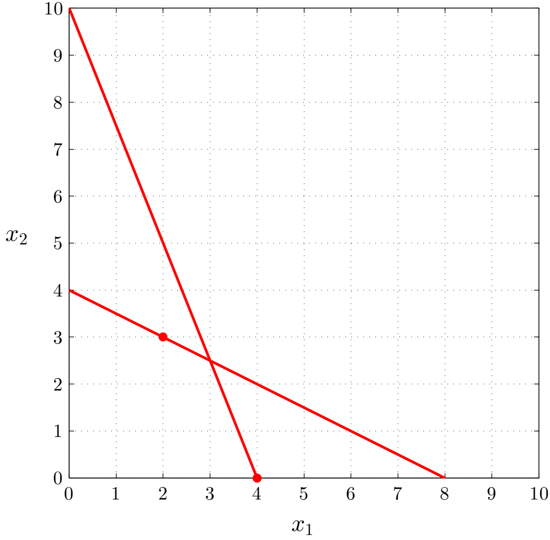

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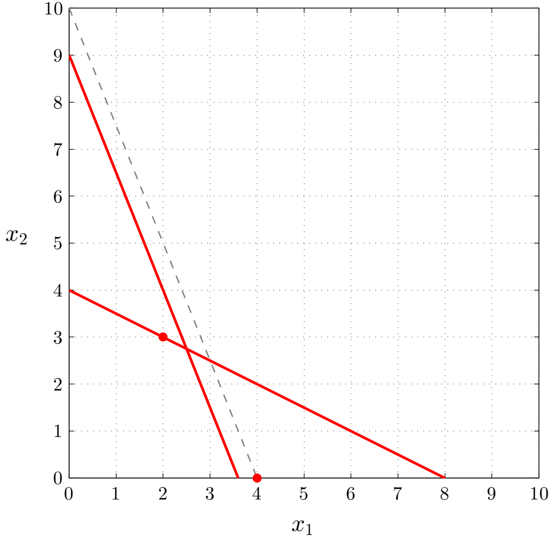

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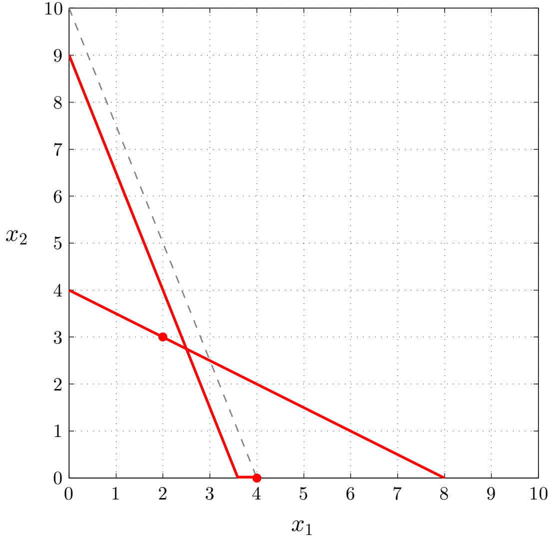

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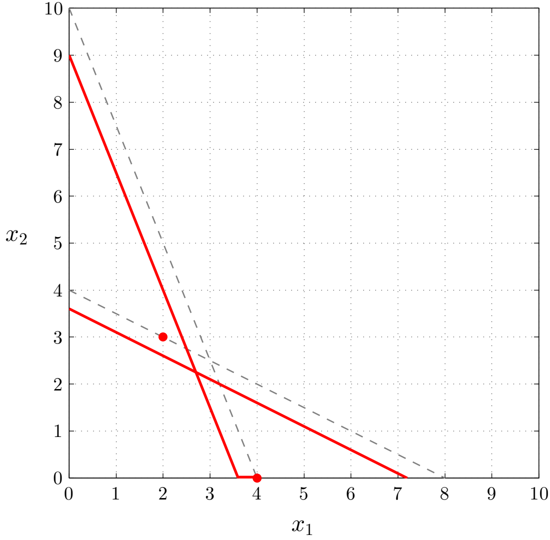

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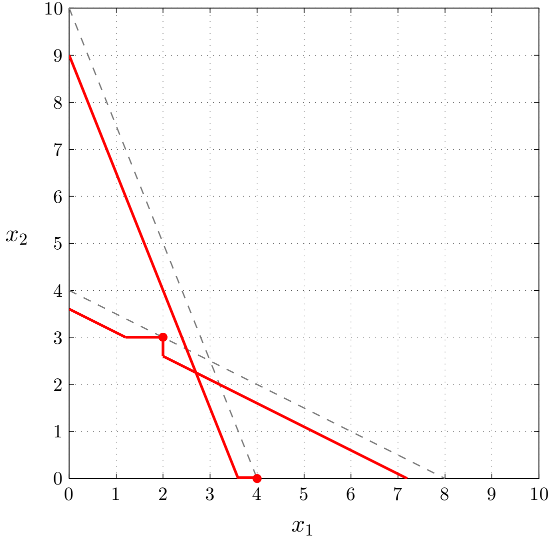

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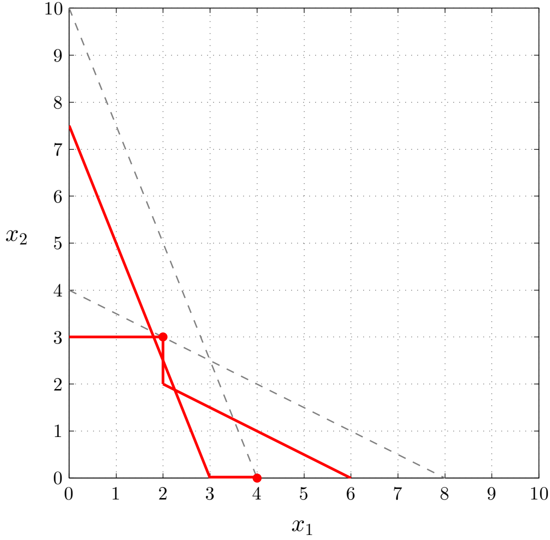

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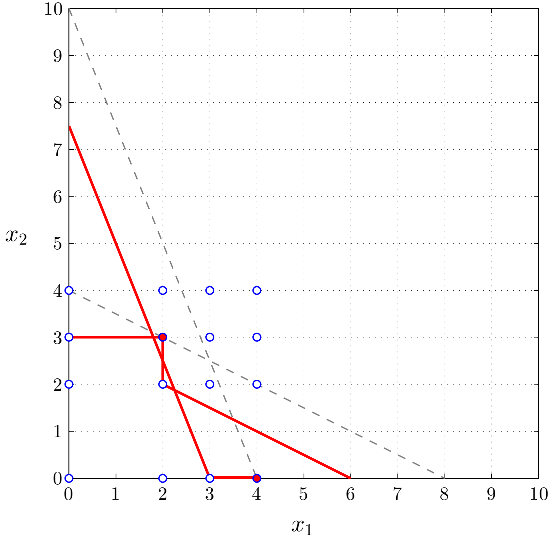

CCEI for Utility Maximization

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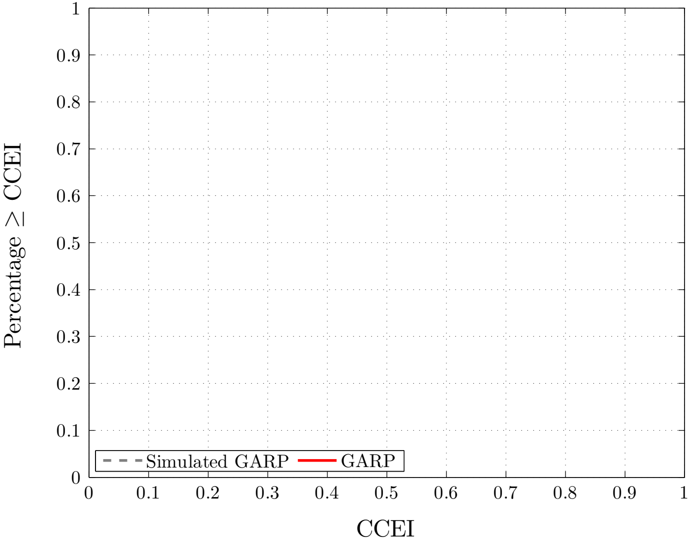

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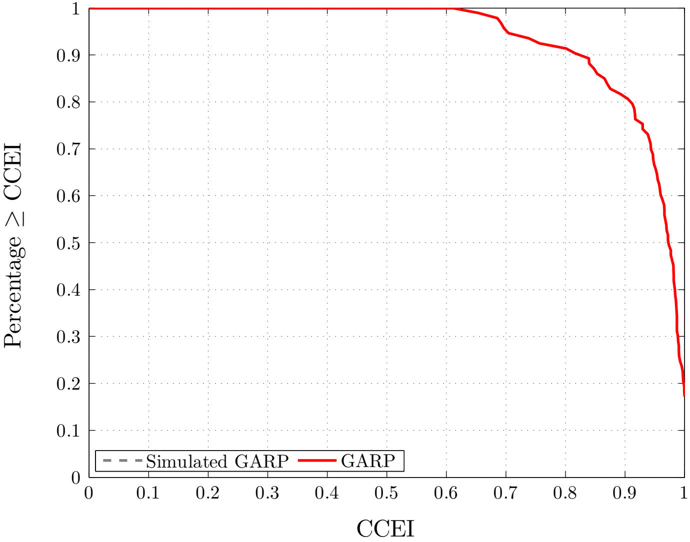

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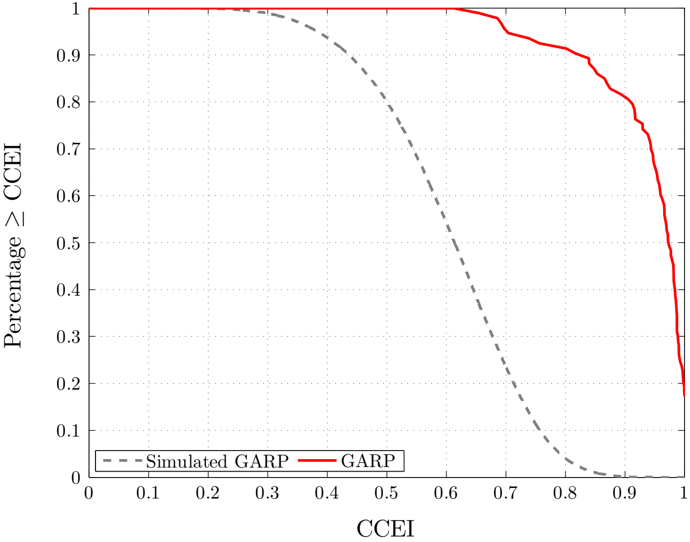

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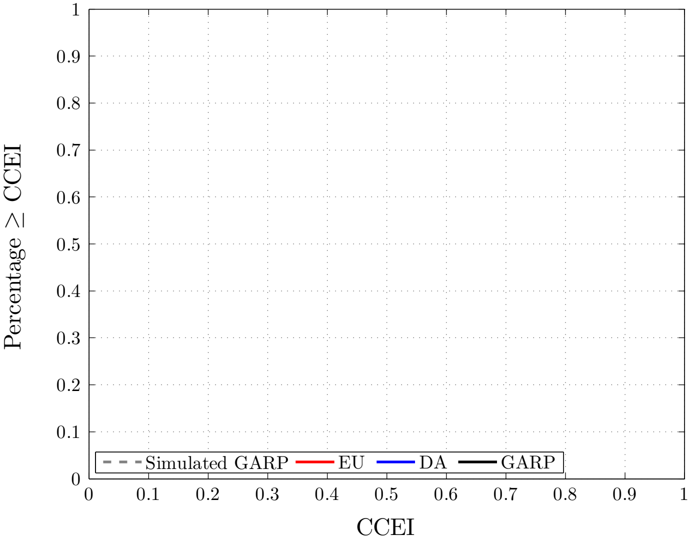

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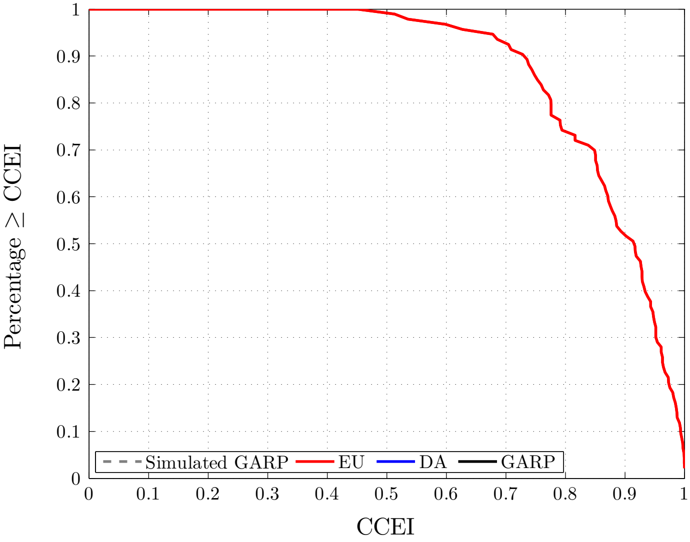

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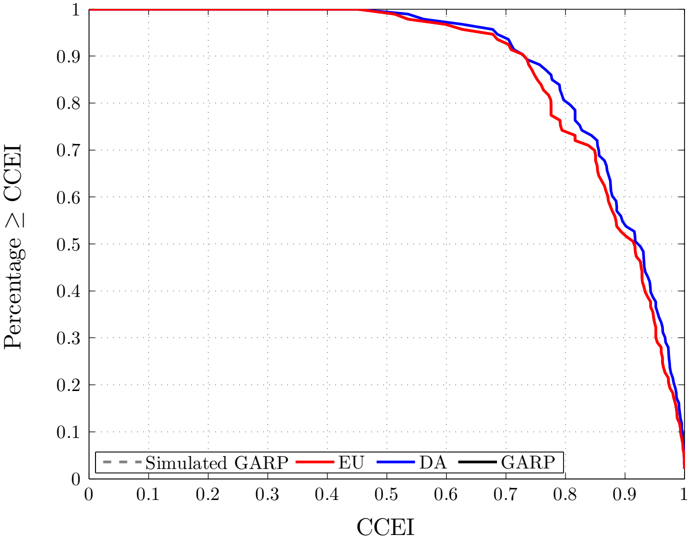

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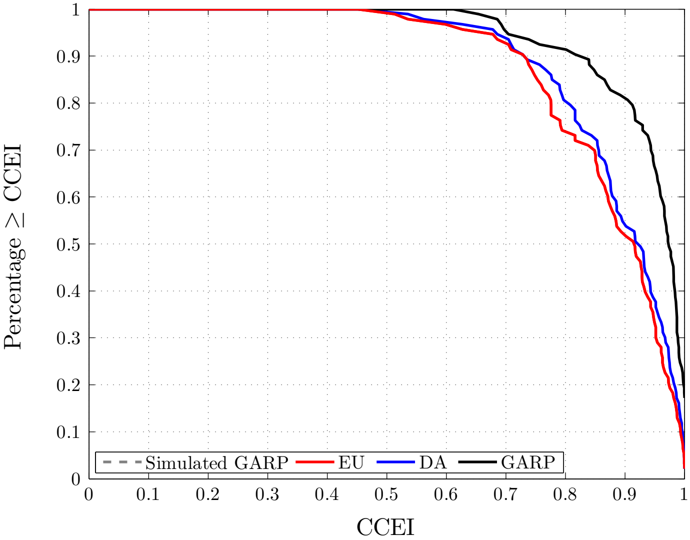

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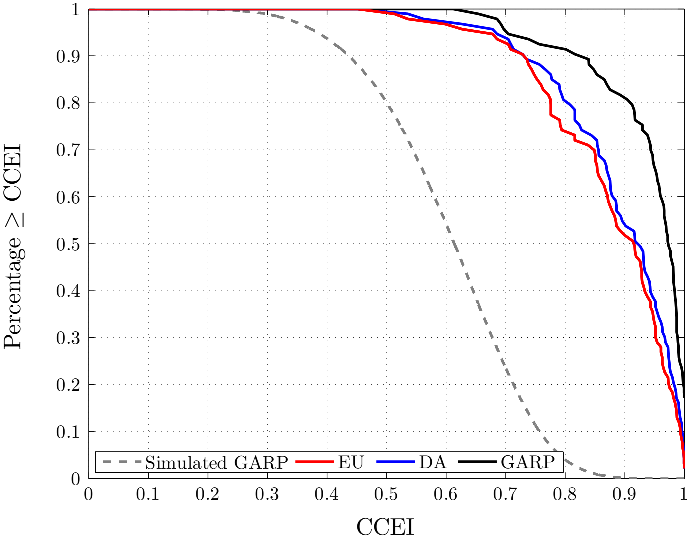

Selten's Index of Predictive Success

Selten (1991) proposes an index of predictive success to measure a model's success in explaining the data. Formally, it is

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The hit rate is the observed frequency of model-consistent outcomes.

The precision is a measure of the size of the set of model-consistent outcomes.

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The precision is a measure of the size of the set of model-consistent outcomes.

A good model is one with a high Hit Rate and a small Precision. In the ideal case, a Hit Rate close to 1 and Precision close to zero. Selten Index ≈ 1.

A poor model is one with a Hit Rate close to zero and Precision close to 1. Selten Index ≈ -1.

The Selten index takes values in $(-1, 1)$. Any model with an index above 0 can be considered to have some predictive success.

Selten's Index of Predictive Success

Selten's index of predictive success = Hit Rate − Precision.

For each subject, we generate 1000 synthetic data sets containing consumption bundles chosen randomly uniformly from the actual budgets facing this subject. (Note: each data set has 50 synthetically generated demand bundles.)

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We take the average of these indices across all subjects to obtain an aggregate Selten index.

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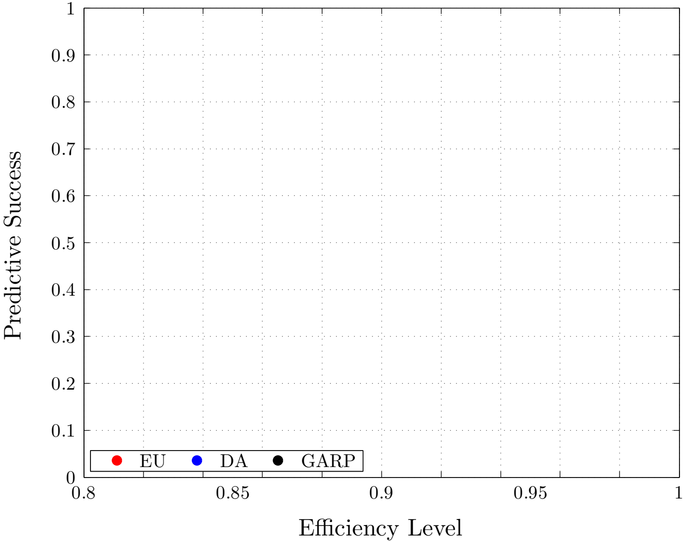

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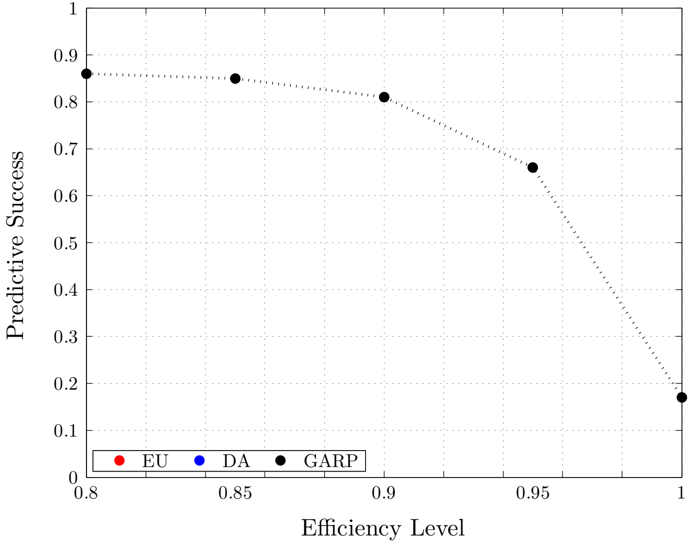

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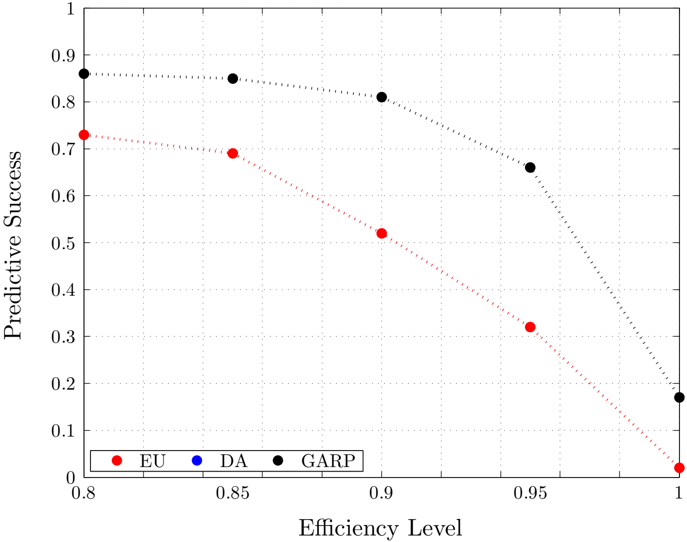

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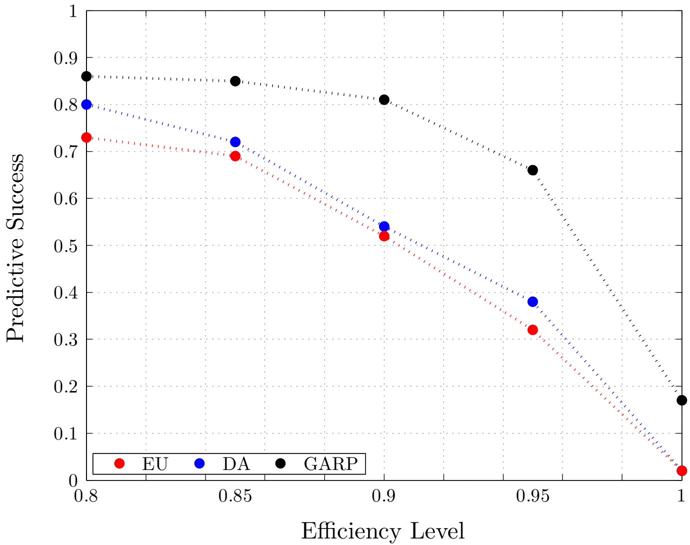

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For each of those subjects who pass GARP at some efficiency level $\bar{\alpha}$, we generate 1000 synthetic data sets that also obey GARP at efficiency level $\bar{\alpha}$. We then calculate, as a measure of precision, the frequency of these data sets being EU or DA-consistent at the same efficiency level.

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Both DA and EU models have very high precision: the probability of a random data set obeying EU/DA, conditional on it obeying GARP, is close to zero.

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The results are unsurprising – all three models are very precise: the probability of a random data set passing the model is almost zero. So the best model is the one with the best hit rate, which is (general) utility-maximization.

But how successful are the EU and DA models in predicting outcomes, *given* that a subject is utility-maximizing?

For each of those subjects who pass GARP at some efficiency level $\bar{\alpha}$, we generate 1000 synthetic data sets that also obey GARP at efficiency level $\bar{\alpha}$. We then calculate, as a measure of precision, the frequency of these data sets being EU or DA-consistent at the same efficiency level.

Both DA and EU models have very high precision: the probability of a random data set obeying EU/DA, conditional on it obeying GARP, is close to zero.

Using this measure of precision, we can calculate the Selten indices for the EU and DA models.

Predictive Success (Conditional on GARP)

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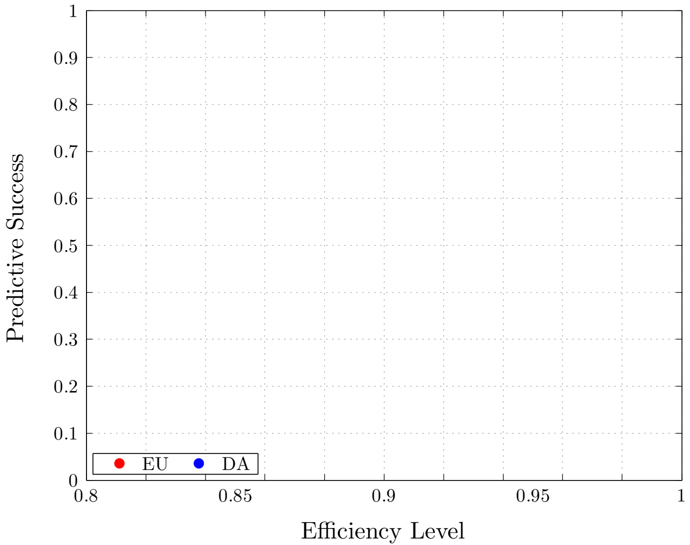
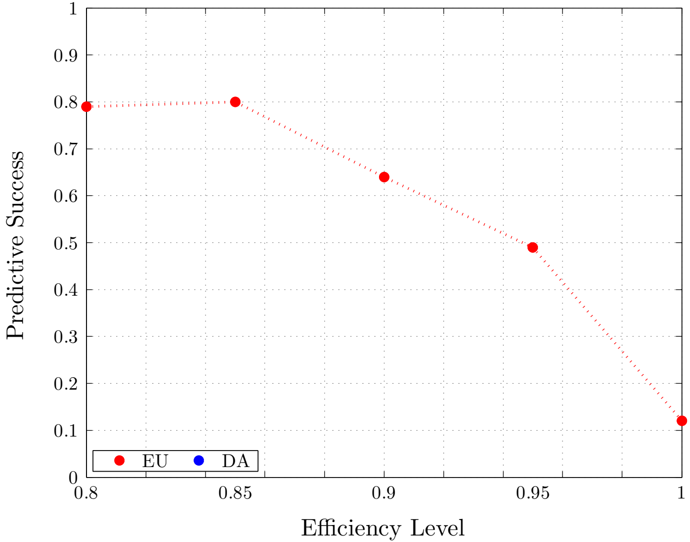


Predictive Success (Conditional on GARP)



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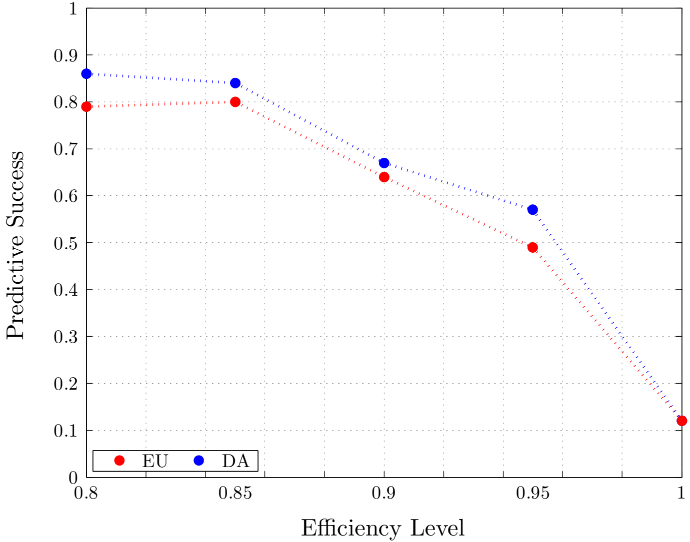

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Our main empirical findings are as follows:

(1) All three models (general utility, expected utility, disappointment aversion) are very precise in the sense that the probability of a randomly drawn data set being consistent with these models is close to zero.

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(4) More than half of the subjects passing GARP are also consistent with the disappointment aversion and expected utility models.

Conclusions

We develop the lattice approach to testing models of decision making under risk and uncertainty.

The approach has the following attractive features:

(1) It avoids ancillary assumptions on the shape of preferences.

(2) It is easy to understand.

(3) It can be easily implemented.

(4) It is flexible enough to measure departures from a model.

(5) It facilitates comparison across models.