Nonparametric Analysis of Monotone Choice

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Introduction

Empirical investigation of consumer demand often uses a nonparametric approach.

Relies on qualitative features of demand derived from revealed preference.

This approach can also be useful for studying games.

We focus on the class of complete information games with strategic complementarity.

Example: IO entry model of airlines

Berry (1992), Ciliberto and Tamer (2009), Kline and Tamer (2016).

Assumptions:

The econometrician observes many markets, defined as trips between two airports.

In each market, there are two airline firms: $i \in \{1, 2\}$

For each market, we observe the covariates (x_1, x_2) and the entry decisions of firms: (N, N), (N, E), (E, N), or (E, E).

Information can be summarized by distribution of firm decisions at realized covariates (x_1, x_2).

		Firm 2	
		N	E
Firm 1	N	P(N,N $\mid x_1, x_2$)	P(N,E $\mid x_1, x_2$)
	E	P(E,N $\mid x_1, x_2$)	P(E,E $\mid x_1, x_2$)

Example: IO entry model of airlines

In Kline and Tamer (2016), covariates are the following:

Market Presence (which is firm-specific) P_1, $P_2 \in \{0, 1\}$ and

Market Size $S \in \{0, 1\}$

Thus all markets can be partitioned into precisely eight types: $(1, 1, 1)$, $(0, 1, 1)$, $(0, 0, 1)$, etc.

In each market, we observe the the entry decision of the pair of firms: (N, N), (N, E), (E, N), or (E, E).

Thus the data consists of precisely eight tables.

		Firm 2	
		N	E
Firm 1	N	P(N,N $\mid x_1, x_2$)	P(N,E $\mid x_1, x_2$)
	E	P(E,N $\mid x_1, x_2$)	P(E,E $\mid x_1, x_2$)

Example: IO entry model of airlines

Distribution of entry choices (from Kline and Tamer (2016))

Covariates = $(0,0,0)$				Covariates = $(0,1,0)$			
N,N	N,E	E,N	E,E	N,N	N,E	E,N	E,E
30.37	68.21	0.55	**0.87**	19	78.51	0.26	2.23
Covariates = $(1,0,0)$				Covariates = $(1,1,0)$			
N,N	N,E	E,N	E,E	N,N	N,E	E,N	E,E
19.38	36.71	25.33	18.58	12.15	54.22	4.99	28.64
Covariates = $(0,0,1)$				Covariates = $(0,1,1)$			
N,N	N,E	E,N	E,E	N,N	N,E	E,N	E,E
15.88	82.28	0.12	1.73	7.80	88.93	0	3.27
Covariates = $(1,0,1)$				Covariates = $(1,1,1)$			
N,N	N,E	E,N	E,E	N,N	N,E	E,N	E,E
10.64	32.64	30.58	26.14	5.53	50.07	2.14	**42.26**

Standard analysis of this data

This involves a parametric specification of payoff functions.

$$\Pi_1(y_1, y_2, x_1, \varepsilon_1) = \begin{cases} \alpha_1' x_1 + \delta_1 1(y_2 = \mathrm{E}) + \varepsilon_1 & \text{if} \quad y_1 = \mathrm{E} \\ 0 & \text{if} \quad y_1 = \mathrm{N} \end{cases}$$

$$\Pi_2(y_1, y_2, x_2, \varepsilon_2) = \begin{cases} \alpha_2' x_2 + \delta_2 1(y_1 = \mathrm{E}) + \varepsilon_2 & \text{if} \quad y_2 = \mathrm{E} \\ 0 & \text{if} \quad y_2 = \mathrm{N} \end{cases}$$

Estimate α_1', δ_1 and α_2', δ_2

Common assumptions:

- interaction effects, δ_1 and δ_2, are negative
- distribution of $(\varepsilon_1, \varepsilon_2)$ belongs to a known family
- $(\varepsilon_1, \varepsilon_2)$ is independent of (x_1, x_2)
- firms play pure strategy Nash equilibrium

Our analysis

We provide a nonparametric test of the joint hypothesis that firms are

(i) playing pure strategy Nash equilibria

(ii) in games of strategic substitutes.

(Note: pure strategy Nash equilibrium always exists in two-player games of substitutes.)

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Features of our approach

- Nonparametric
- No assumption on equilibrium selection mechanism
- Very general approach to model heterogeneity
  - no assumption on the joint distribution of $(\varepsilon_1, \varepsilon_2)$
  - no assumption on group formation

But we do assume $(\varepsilon_1, \varepsilon_2)$ is independent of (x_1, x_2).
In other words, we require the unobserved heterogeneity to be distributed independently of the covariates.

How does the idea work?

Key observation: we should focus on behavior and not payoffs.

A realization of $(\varepsilon_1, \varepsilon_2)$ leads to $\Pi_1(y_1, y_2, x_1, \varepsilon_1)$ and $\Pi_2(y_1, y_2, x_2, \varepsilon_2)$.

Combined with an equilibrium selection rule, these payoffs induce a sequence of Nash equilibrium choices across different covariates (x_1, x_2).

This sequence of joint choices across covariates correspond to a behavioral type.

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This sequence of joint choices across covariates correspond to a behavioral type.

Data can only convey information up to behavioral types – nothing finer:

Distinct values of $(\varepsilon_1, \varepsilon_2)$ are indistinguishable if they generate the same behavioral type.

Note:

Each value of $(\varepsilon_1, \varepsilon_2)$ can lead to more than one behavioral type because of equilibrium selection.

How does the idea work?

Suppose that firm i's payoff is
(i) increasing with x_i and (ii) falls with entry of the other firm.

This will restrict the set of behavioral types. Only certain types are consistent with these sign restrictions.

Assume x_1 is fixed and x_2 takes three values: $(0, 0)$, $(0, 1)$ and $(1, 0)$.

(E, E) at $(0, 0)$, (E, N) at $(0, 1)$, and (N, N) at $(1, 0)$ is impossible.

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(E, N) at $(0, 0)$, (N, E) at $(0, 1)$ and (E, E) at $(1, 0)$ is impossible.

As possible joint choices and covariate values are finite, we can enumerate ALL consistent behavioral types.

How does the idea work?

We observe the distribution across joint actions at $x_2 = (0,0)$, $(0,1)$, and $(1,0)$. (x_1 held fixed.)

		Firm 2	
		N	E
Firm 1	N	$P(N,N \mid x_1, x_2)$	$P(N,E \mid x_1, x_2)$
	E	$P(E,N \mid x_1, x_2)$	$P(E,E \mid x_1, x_2)$

We want to test the hypothesis that firms

(i) are playing pure strategy Nash equilibria;
(ii) have payoff functions with given sign restrictions.

First, we find all behavioral types consistent with (i) and (ii).

The data is consistent with our hypothesis if we can find a distribution over consistent behavioral types that explain the observations.

How does the idea work?

Suppose we have the following data (for some fixed x_1)

$x_2 = (0,0)$		Firm 2	
		N	E
Firm 1	N	3/12	3/12
	E	4/12	2/12

$x_2 = (0,1)$		Firm 2	
		N	E
Firm 1	N	1/12	5/12
	E	3/12	3/12

$x_2 = (1,0)$		Firm 2	
		N	E
Firm 1	N	2/12	4/12
	E	2/12	4/12

How does the idea work?

Data can be rationalized by the following consistent types.

Type	Weight	$x_2 = (0,0)$				$x_2 = (0,1)$				$x_2 = (1,0)$			
		Action profiles				Action profiles				Action profiles			
		N,N	N,E	E,N	E,E	N,N	N,E	E,N	E,E	N,N	N,E	E,N	E,E
1				●					●			●	
2		●					●			●			
3				●				●					●
4				●				●				●	
5		●				●					●		
6					●				●				●
7			●				●				●		

How does the idea work?

Data can be rationalized as follows.

Type	Weight	$x_2 = (0,0)$				$x_2 = (0,1)$				$x_2 = (1,0)$			
		Action profiles				Action profiles				Action profiles			
		N,N	N,E	E,N	E,E	N,N	N,E	E,N	E,E	N,N	N,E	E,N	E,E
1	1/12			1/12					1/12			1/12	
2	2/12	2/12					2/12			2/12			
3	2/12			2/12				2/12					2/12
4	1/12			1/12				1/12				1/12	
5	1/12	1/12				1/12					1/12		
6	2/12				2/12				2/12				2/12
7	3/12		3/12				3/12				3/12		
Sum	1	3/12	3/12	4/12	2/12	1/12	5/12	3/12	3/12	2/12	4/12	2/12	4/12

$x_2 = (0,0)$		Firm 2	
		N	E
Firm 1	N	3/12	3/12
	E	4/12	2/12

$x_2 = (0,1)$		Firm 2	
		N	E
Firm 1	N	1/12	5/12
	E	3/12	3/12

$x_2 = (1,0)$		Firm 2	
		N	E
Firm 1	N	2/12	4/12
	E	2/12	4/12

Our general approach

Step 1

Determine ALL the behavioral types that are consistent with the sign restrictions on payoff functions.

The paper formulates a Revealed Monotonicity Axiom:

can be used to check for consistency of a behavioral type with pure strategy Nash equilibrium play in games of strategic complements.

If a behavioral type obeys the axiom, then it is possible to construct payoff functions that

(i) obey single crossing conditions and

(ii) have observed actions as optimal.

Our general approach

Step 1

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The Revealed Monotonicity Axiom provides a procedure to do this.

Step 2

Determine whether or not there is a distribution over the consistent types identified in Step 1 that explains the data.

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Our general approach

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Determine ALL the behavioral types that are consistent with the sign restrictions on payoff functions.

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Step 2

Determine whether or not there is a distribution over the consistent types identified in Step 1 that explains the data.

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If data set passes the test, we can perform further analyses. For example

(i) estimate the fraction of types of with specific features or

(ii) make estimates of the distribution of equilibrium behavior at out-of-sample covariate values.

Distribution of entry choices in the data

Data set is from Kline and Tamer (2016). Covariates are the following:

Market Presence (which is firm-specific) $P_1, P_2 \in \{0, 1\}$ and

Market Size $S \in \{0, 1\}$

Thus all markets can be partitioned into precisely eight types.

Covariates = $(0,0,0)$				Covariates = $(0,1,0)$			
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10.64	32.64	30.58	26.14	5.53	50.07	2.14	**42.26**

Hypothesis Testing

Hypothesis:

This data set is generated by pure strategy Nash equilibria where firm i's payoff has the following properties:

(i) it is increasing in its market presence P_i and in market size S;
(ii) its payoff when it enters is strictly higher if the other firm stays out.

There are four possible joint choices for each of the eight covariate values.

Thus there are $4^8 = 65,536$ behavioral types.

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Thus there are $4^8 = 65,536$ behavioral types.

Of these, 482 are consistent with hypothesis

	$(0,0,0)$	$(0,1,0)$	$(1,0,0)$	...	$(1,1,1)$	
type 1	N,N	E,E	E,E	...	N,N	RM axiom ✓
type 2	E,E	E,E	E,E	...	N,E	RM axiom $\times$
...	...	...		...	...	...

Stochastic test

Testing the hypothesis requires checking whether a system of linear equations

$$Ax = b$$

$$A \text{ is } 32 \times 482, \ x \text{ is } 482 \times 1, \text{ and } b \text{ is } 32 \times 1$$

has a *positive* solution in x.

- A is a matrix of 0's and 1's that captures all consistent types
  - each column captures the behavior of a type under all covariate values
- b is the observed distribution of entry choices
- x is a possible distribution of behavioral types

Empirical results

- Data is not consistent with our hypothesis
- At least one violation is

$$P(N,N|1,1,0) + P(E,N|1,1,0) = 17.14\% < 19\% = P(N,N|0,1,0)$$

Closest compatible choice distribution

It is quite similar!

Covariates = $(0,0,0)$				Covariates = $(0,1,0)$			
N,N	N,E	E,N	E,E	N,N	N,E	E,N	E,E
30.06	67.90	0.86	1.18	18.29	78.96	0.71	2.05
Covariates = $(1,0,0)$				Covariates = $(1,1,0)$			
N,N	N,E	E,N	E,E	N,N	N,E	E,N	E,E
19.38	36.71	25.33	18.58	12.73	53.64	5.57	28.06
Covariates = $(0,0,1)$				Covariates = $(0,1,1)$			
N,N	N,E	E,N	E,E	N,N	N,E	E,N	E,E
15.46	81.86	0.54	2.15	7.86	89.19	0.26	2.69
Covariates = $(1,0,1)$				Covariates = $(1,1,1)$			
N,N	N,E	E,N	E,E	N,N	N,E	E,N	E,E
10.64	32.64	30.58	26.14	5.63	49.98	2.24	42.16

Small sample considerations

Kitamura and Stoye (2017)

The Null-Hypothesis is

$$(\mathrm{H}) \quad \min_{x \in \mathbb{R}_+^{482}} (b - Ax)' (b - Ax) = 0.$$

The sample counterpart is

$$J_N = N \min_{x \in \mathbb{R}_+^{482}} \left(\hat{b} - Ax\right)' \left(\hat{b} - Ax\right) = 0.$$

p-value is about 15%.

Thus, we don't reject the Null-Hypothesis.

Application: Estimating non-strategic types

Out of the 482 consistent types, 36 are also consistent with non-strategic interactions, i.e.,

the entry decision of both airlines can depend on covariates but not on the entry decision of the other airline

We can estimate bounds for the set of 36 non-strategic types

Upper bound is 75%

Conclusion: strategic interaction is crucial to explaining the data.

Concluding Remarks

- We provide a nonparametric test of strategic complementarity in games. Applicable to
  - time-series data
  - cross-sectional data
- The procedure can be developed to make out-of-sample predictions of equilibrium behavior.
- The procedure can be easily implemented.