A Theory of Revealed Indirect Preference

Gaoji Hu, Jiangtao Li, John K.-H. Quah, and Rui Tang

April 18, 2022

Preference over Menus

Let X be a set of alternatives and A and B subsets of X, which we interpret as menus.

The observer knows the agent's preference between menus A and B:

either the agent strictly prefers A over B, or prefers A over B, or is indifferent.

The observer collects a data set of menu preference pairs

$$\mathcal{M} := \{(A^t, B^t)\}_{t \in T},$$

where T is the disjoint union of S and W.

If $t \in S$, then $A^t \succ B^t$ and if $t \in W$ then $A^t \succsim B^t$.

When is this preference over menus generated by a preference over the alternatives in X?

Rationalizing a preference over menus

A preference $\succsim$ over the elements in X is a complete and transitive binary relation.

Definition. Let $\mathcal{M} := \{(A^t, B^t\}_{t \in T}$ be a set of menu preference pairs.

A preference $\succsim$ on X rationalizes $\mathcal{M}$ if

there is $x^t \in A^t$ such that $x^t \succsim y$ for all $y \in B^t$ $(t \in T)$ and

$x^t \succ y$ for all $y \in B^t$ if $t \in S$.

Given a data set $\mathcal{M}$, how do we check if $\mathcal{M}$ is rationalizable, i.e., that $\succsim$ exists?

Note: as stated, the question is non-trivial only if S is nonempty.

Rationalizing a preference over menus

Suppose that $T = S$, so $A^t \succ B^t$ for all $t \in T$.

Suppose that $\succsim$ rationalizes $\mathcal{M}$ so there exists $x^t \succ B^t$ for all t.

Fix $T' \subseteq T$. Then there is some $t^* \in T'$, we have $x^{t^*} \succsim x^t$ for all $t \in T'$.

Then $x^{t^*} \succ y$ for all $y \in B(T') := \cup_{t \in T'} B^t$.

In particular, $x^{t^*} \notin B(T')$, so

$$A(T') \setminus B(T') \text{ is nonempty.}$$

$\mathcal{M}$ satisfies the partial congruence axiom if this property holds for all $T' \subseteq T$.

Rationalizing a preference over menus

Suppose that $T = S$, so $A^t \succ B^t$ for all $t \in T$.

Suppose that $\succsim$ rationalizes $\mathcal{M}$ so there exists $x^t \succ B^t$ for all t.

Fix $T' \subseteq T$. Then there is some $t^* \in T'$, we have $x^{t^*} \succsim x^t$ for all $t \in T'$.

Then $x^{t^*} \succ y$ for all $y \in B(T') := \cup_{t \in T'} B^t$.

In particular, $x^{t^*} \notin B(T')$, so

$$A(T') \setminus B(T') \text{ is nonempty.}$$

$\mathcal{M}$ satisfies the partial congruence axiom if this property holds for all $T' \subseteq T$.

Theorem. (Fishburn, 1976)

$\mathcal{M} := \{(A^t, B^t\}_{t \in T}$, with $T = S$ can be rationalized if and only if $\mathcal{M}$ satisfies the partial congruence axiom.

Rationalizing a preference over menus

Example.

$A^1 = \{x, y\}$ $B^1 = \{r, w\}$;

$A^2 = \{r, y\}$ $B^2 = \{z, w\}$;

$A^3 = \{x, w\}$ $B^3 = \{r, y\}$.

Rationalization is possible with $x \succ y \succ r \succ w \succ z$.

Partial congruence axiom: $A(T') \setminus B(T')$ is nonempty for all nonempty $T' \subseteq T$.

Rationalizing a preference over menus

Example.

$A^1 = \{x, y\}\ \ B^1 = \{r, w\};$

$A^2 = \{r, y\}\ \ B^2 = \{z, w\};$

$A^3 = \{x, w\}\ \ B^3 = \{r, y\}.$

Rationalization is possible with $x \succ y \succ r \succ w \succ z$.

Partial congruence axiom: $A(T') \setminus B(T')$ is nonempty for all nonempty $T' \subseteq T$.

$A^1 = \{x, y\}\ \ B^1 = \{r, w\};$

$A^2 = \{r, y\}\ \ B^2 = \{x, z\};$

$A^3 = \{x, w\}\ \ B^3 = \{r, y\}.$

Rationalization is impossible. Indeed $(\cup_{i=1}^3 A^i) \setminus (\cup_{i=1}^3 B^i)$ is empty.

Rationalizing a preference over menus

But notice that

$A^1 = \{x, y\}$ $B^1 = \{r, w\}$;

$A^2 = \{r, y\}$ $B^2 = \{x, z\}$;

$A^3 = \{x, w\}$ $B^3 = \{r, y\}$.

can be rationalized if we require $A^1 \succ B^1$, $A^2 \succsim B^2$, and $A^3 \succsim B^3$.

Then $x \sim y \succ r \sim w \succ z$ rationalizes data.

So the partial congruence axiom isn't the right property to check if agent reports a weak preference between menus.

Rationalizing a preference over menus

Our objective is to generalize Fishburn's result in multiple directions.

1. What if there is a weak preference between menus, i.e., $A^t \succsim B^t$?

Rationalizing a preference over menus

Our objective is to generalize Fishburn's result in multiple directions.

1. What if there is a weak preference between menus, i.e., $A^t \succsim B^t$?

2. What if we are in an environment where there is a natural order on alternatives that the preference should respect and/or when there is a natural topology?

For example, in classical consumer theory...

Rationalizing a preference over menus

Our objective is to generalize Fishburn's result in multiple directions.

1. What if there is a weak preference between menus, i.e., $A^t \succsim B^t$?

2. What if we are in an environment where there is a natural order on alternatives that the preference should respect and/or when there is a natural topology?

For example, in classical consumer theory...

3. Testing the partial congruence axiom as stated is not viable: it requires checking if $A(T') \setminus B(T')$ is nonempty for all possible sets $T' \subseteq T$.

Is there an efficient algorithm for testing this axiom?

Rationalizing a preference over menus

Our objective is to generalize Fishburn's result in multiple directions.

1. What if there is a weak preference between menus, i.e., $A^t \succsim B^t$?

2. What if we are in an environment where there is a natural order on alternatives that the preference should respect and/or when there is a natural topology?

For example, in classical consumer theory...

3. Testing the partial congruence axiom as stated is not viable: it requires checking if $A(T') \setminus B(T')$ is nonempty for all possible sets $T' \subseteq T$.

Is there an efficient algorithm for testing this axiom?

4. Apply results to newer models of choice.

Never-covered property

Let $\mathcal{M} = \{(A^t, B^t)\}_{t \in T}$ where $T = W \cup S$ and S is nonempty.

Iterated deletion of strictly dominated observations:

let $T' \subseteq T$ and $S' = T' \cap S$; define

$$\Phi^1(T') = \left\{ t \in T' : A^t \subseteq B(S') \right\}$$
$$\Phi^2(T') = \left\{ t \in T' : A^t \subseteq B\left(S' \cup \Phi^1(T')\right) \right\}$$
$$\Phi^3(T') = \left\{ t \in T' : A^t \subseteq B\left(S' \cup \Phi^2(T')\right) \right\} \text{ and so on.}$$

Notice that $\Phi^1(T') \subseteq \Phi^2(T') \subseteq \Phi^3(T') \subseteq \Phi^4(T') \ldots$

Eventually, we obtain $\Phi^m(T') = \Phi^{m+1}(T')$. Then we stop and define $\Phi(T') = \Phi^m(T')$.

Never-covered property

Let $\mathcal{M} = \{(A^t, B^t)\}_{t\in T}$ where $T = W \cup S$ and S is nonempty.

Iterated deletion of strictly dominated observations:

let $T' \subseteq T$ and $S' = T' \cap S$; define

$$\Phi^1(T') = \left\{t \in T' : A^t \subseteq B(S')\right\}$$
$$\Phi^2(T') = \left\{t \in T' : A^t \subseteq B\left(S' \cup \Phi^1(T')\right)\right\}$$
$$\Phi^3(T') = \left\{t \in T' : A^t \subseteq B\left(S' \cup \Phi^2(T')\right)\right\} \text{ and so on.}$$

Notice that $\Phi^1(T') \subseteq \Phi^2(T') \subseteq \Phi^3(T') \subseteq \Phi^4(T') \ldots$

Eventually, we obtain $\Phi^m(T') = \Phi^{m+1}(T')$. Then we stop and define $\Phi(T') = \Phi^m(T')$.

Suppose $\mathcal{M}$ is rationalized by $\succsim$ so there is $t^* \in T'$ and $x^{t^*} \in A^{t^*}$ such that $x^{t^*} \succsim B(T')$ and $x^{t^*} \succ B(S')$.

Then $t^* \notin \Phi^1(T')$ because $x^{t^*} \notin B(S')$.

Never-covered property

Let $\mathcal{M} = \{(A^t, B^t)\}_{t \in T}$ where $T = W \cup S$ and S is nonempty.

Iterated deletion of strictly dominated observations:

let $T' \subseteq T$ and $S' = T' \cap S$; define

$$\Phi^1(T') = \left\{t \in T' : A^t \subseteq B(S')\right\}$$
$$\Phi^2(T') = \left\{t \in T' : A^t \subseteq B\left(S' \cup \Phi^1(T')\right)\right\}$$
$$\Phi^3(T') = \left\{t \in T' : A^t \subseteq B\left(S' \cup \Phi^2(T')\right)\right\} \text{ and so on.}$$

Notice that $\Phi^1(T') \subseteq \Phi^2(T') \subseteq \Phi^3(T') \subseteq \Phi^4(T') \ldots$

Eventually, we obtain $\Phi^m(T') = \Phi^{m+1}(T')$. Then we stop and define $\Phi(T') = \Phi^m(T')$.

Suppose $\mathcal{M}$ is rationalized by $\succsim$ so there is $t^* \in T'$ and $x^{t^*} \in A^{t^*}$ such that $x^{t^*} \succsim B(T')$ and $x^{t^*} \succ B(S')$.

Then $t^* \notin \Phi^1(T')$ because $x^{t^*} \notin B(S')$. Furthermore,

$t^* \notin \Phi^2(T') \ldots$, $t^* \notin \Phi(T')$. Thus $T' \setminus \Phi(T')$ is nonempty.

Never-covered property

Definition. $\mathcal{M} = \{(A^t, B^t)\}_{t \in T}$ satisfies the never-covered property if

$$T' \setminus \Phi(T') \textit{ is nonempty for all nonempty } T' \subset T.$$

This notion generalizes the partial congruence axiom.

Theorem. A data set of menu preference pairs $\mathcal{M} = \{(A^t, B^t)\}_{t \in T}$ (with $T = W \cup S$) is rationalizable if and only if it satisfies the never-covered property.

Never-covered property

Definition. $\mathcal{M} = \{(A^t, B^t)\}_{t \in T}$ satisfies the never-covered property if

$$T' \setminus \Phi(T') \text{ is nonempty for all nonempty } T' \subset T.$$

This notion generalizes the partial congruence axiom.

Theorem. A data set of menu preference pairs $\mathcal{M} = \{(A^t, B^t)\}_{t \in T}$ (with $T = W \cup S$) is rationalizable if and only if it satisfies the never-covered property.

An algorithm?

Algorithm for checking the never-covered property

First calculate $\Phi(T)$ and let $T^1 = \Phi(T)$.

Define $T^{n+1} := \Phi(T^n)$.

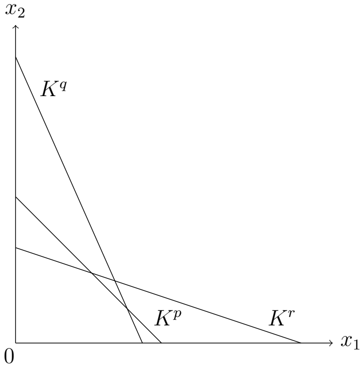
We have $T \supseteq T^1 \supseteq T^2 \supseteq T^3 \supseteq T^4 \supseteq \ldots$.

One of two possibilities will occur:

either we obtain m such that $T^{m+1} := \Phi(T^m) = T^m$ in which the never-covered property fails

or, we obtain m such that $T^{m+1} := \Phi(T^m)$ is empty, in which case the never-covered property holds.

Menus in consumption space $\mathbb{R}^\ell_+$



Can $K^p \succsim K^q$ and $K^q \succsim K^r$?

NO, provided preference is increasing with consumption. Reason:

if it holds, then there is $x \in K^p$ such that $x \succsim y$ for all $y \in K^q \cup K^r$.

But impossible because every bundle in K^p is strictly inferior to something in K^q or K^r.

Menus in consumption space

The never-covered property can be modified to incorporate various requirements on the rationalizing preference/utility function.

Consider the case where $X = \mathbb{R}^\ell_+$, the standard ℓ-good consumption space.

The data set is $\mathcal{M} = \{(A^t, B^t)\}_{t \in T}$, where A^t, $B^t \subset \mathbb{R}^\ell_+$.

Example. A^t and B^t are linear budget sets.

$$A^t = \{x \in \mathbb{R}^\ell_+ : p^t \cdot x \leq 1\}$$

where $p^t = (p^t_1, p^t_2, \ldots, p^t_\ell) \gg 0$. (Similarly for B^t.)

Question. When can we rationalize $\mathcal{M}$ with a strictly increasing preference $\succsim$, i.e., a preference $\succsim$ where $x'' \succ x'$ if $x'' > x'$?

Menus in consumption space

Given a data set $\mathcal{M} = \{(A^t, B^t)\}_{t \in T}$, we can define, for any $T' \subseteq T$,

$\Phi_{\geq}(T')$, *the set of dominated observations in* T'

(which now depends on the product order $\geq$).

Definition. $\mathcal{M}$ satisfies the never-covered property under $\geq$ if $T' \setminus \Phi_{\geq}(T')$ is nonempty for every nonempty $T' \subseteq T$.

Theorem. The following statements on $\mathcal{M}$ are equivalent:

(1) $\mathcal{M}$ is rationalizable by a strictly increasing preference.

(2) $\mathcal{M}$ satisfies the never-covered property under $\geq$.

(3) $\mathcal{M}$ is rationalizable by a preference that admits a strictly increasing and continuous utility function.

Application 1: Rationalizing price preferences

Suppose that $A^t = L(p^t)$ and $B^t = L(q^t)$ – so all budget sets are linear and can be represented by prices (with income normalized at 1).

Let $\mathcal{M} = \left\{(L(p^t), L(q^t))\right\}_{t\in T}$ be a data set of price preferences.

Corollary. The following statements on $\mathcal{M}$ are equivalent.

(1) $\mathcal{M}$ can be rationalized by a strictly increasing preference.

(2) $\mathcal{M}$ satisfies the never-covered property under $\geq$.

(3) $\mathcal{M}$ can be nicely rationalized by a strictly increasing, continuous, and concave utility function.

This result is the finite analog to the classical result: a function $v : \mathbb{R}^\ell_{++} \to \mathbb{R}$ is a bona fide indirect utility function, i.e., there exists u such that $v(p) = \max\{u(x) : x \in L(p)\}$, if and only v is quasi-convex.

Menus in consumption space

Iterated accumulation of strictly dominated observations:

let $T' \subseteq T$, $W' = T' \cap W$, $S' = T' \cap S$; define

$$\Phi^1_{\geq}(T') = \left\{t \in T' : A^t \subseteq B(W')^0 \cup B(S')\right\}$$
$$\Phi^2_{\geq}(T') = \left\{t \in T' : A^t \subseteq B(W')^0 \cup B(S' \cup \Phi^1_{\geq}(T'))\right\}$$
$$\Phi^3_{\geq}(T') = \left\{t \in T' : A^t \subseteq B(W')^0 \cup B(S' \cup \Phi^2_{\geq}(T'))\right\}$$

and so on until $\Phi^{m+1}_{\geq}(T') = \Phi^m_{\geq}(T')$, in which case define

$$\Phi_{\geq}(T') := \Phi^m_{\geq}(T').$$

Straightforward to check: if $\mathcal{M}$ is rationalized by a strictly increasing preference $\succsim$, then never-covered property holds, i.e.,

$$T' \setminus \Phi_{\geq}(T') \text{ is nonempty.}$$

Application 2: Generalizing Afriat's Theorem

Suppose we observe a consumer choosing a bundle x^t from the budget set

$$L(p^t) = \{x \in \mathbb{R}^\ell_+ : p^t \cdot x \leq 1\}.$$

The data set has the form $\mathcal{O} = \{(x^t, L(p^t))\}_{t \in T}$.

Question posed by Afriat: when can we find a strictly increasing utility function $u : \mathbb{R}^\ell_+ \to \mathbb{R}$ such that

$$u(x^t) \geq u(x) \quad \text{for all } x \in L(p^t)$$

Afriat's Theorem. $\mathcal{O} = \{(x^t, L(p^t))\}_{t \in T}$ can be rationalized by a strictly increasing utility function if and only if it satisfies the generalized axiom of revealed preference (GARP).

Application 2: Generalizing Afriat's Theorem

Suppose that the observer does not observe the choice exactly but only knows that it falls within some set

$$A^t \subseteq L(p^t).$$

So he has access to a coarse data set $\mathcal{O} = \{(A^t, L(p^t))\}_{t \in T}$.

Then the natural question is the following:

when does there exist a strictly increasing utility function u and $x^t \in A^t$ for each $t \in T$ such that $u(x^t) \geq u(x)$ for all $x \in L(p^t)$?

u exists if and only if $\mathcal{O} = \{(A^t, L(p^t))\}_{t \in T}$, with $T = W$, satisfies the never-covered property.

Application 2: Generalizing Afriat's Theorem

Afriat's Theorem, generalized.

Let $\mathcal{O} = \{(A^t, L(p^t))\}_{t \in T}$ be a coarse data set. Then the following statements are equivalent.

(1) $\mathcal{O}$ can be rationalized by a strictly increasing preference.

(2) $\mathcal{O}$ satisfies the never-covered property under $\geq$.

(3) $\mathcal{O}$ can be rationalized by a preference with a utility representation that is strictly increasing, continuous and concave.

Application of generalized Afriat's Theorem

It is common for data sets $\mathcal{O} = \{(x^t, L(p^t))\}_{t \in T}$ to violate GARP.

There are various indices devised to measure the 'seriousness' of the violation.

An obvious way of doing this is to check the extent to which we need to perturb x^t so that the perturbed choice $\tilde{x}^t$ is such that

$$\{(\tilde{x}^t, L(p^t)\}_{t \in T}$$

can be rationalized.

$\mathcal{O}$ is close to being rational if $\tilde{x}^t$ is not far from x^t.

The perturbation index

Let $A^t_\kappa = \{x \in L(p^t) : \; p^t \cdot x = 1 \text{ and } |p^t_i x_i - p^t_i x^t_i| \leq \kappa \text{ for all } i\}$,

where $\kappa \in [0, 1]$.

Is $\mathcal{O}_\kappa = \{(A^t_\kappa, L(p^t))\}_{t \in T}$ rationalizable?

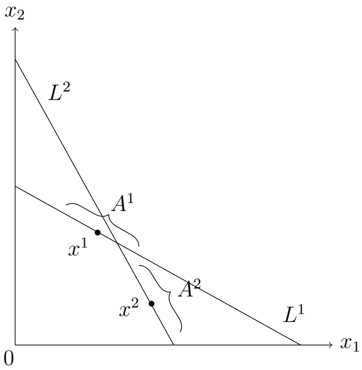
In other words, is there $\tilde{x}^t \in A^t_\kappa$ and an increasing utility function u such that $u(\tilde{x}^t) \geq u(x)$ for all $x \in L(p^t)$.

$\mathcal{O}_\kappa$ must be rationalizable for κ sufficiently close to 1.

Definition. The perturbation index of $\mathcal{O} = \{(x^t, L(p^t))\}_{t \in T}$ is

$$\kappa^* = \min\{\kappa : \mathcal{O}_\kappa \text{ is rationalizable by an increasing preference}\}.$$

The perturbation index



Original data set $\{(x^1, L^1), (x^2, L^2)\}$ is not rationalizable.

But $\{(A^1, L^1), (A^2, L^2)\}$ is rationalizable.

κ^* is the smallest value so that $A^1 \not\subseteq L^2$.

Algorithm for checking the never-covered property

First calculate $\Phi(T)$ and let $T^1 = \Phi(T)$.

Define $T^{n+1} := \Phi(T^n)$.

We have $T \supseteq T^1 \supseteq T^2 \supseteq T^3 \supseteq T^4 \supseteq \ldots$.

One of two possibilities will occur:

either we obtain m such that $T^{m+1} := \Phi(T^m) = T^m$ in which the never-covered property fails

or, we obtain m such that $T^{m+1} := \Phi(T^m)$ is empty, in which case the never-covered property holds.

Computing perturbation indices

	$\kappa = 0.2$	$\kappa = 0.3$
$\lvert T\rvert$	50	50
$\lvert T^1\rvert$	48	47
$\lvert T^2\rvert$	47	46
$\lvert T^3\rvert$	45	44
$\lvert T^4\rvert$	41	39
$\lvert T^5\rvert$	39	36
$\lvert T^6\rvert$	37	31
$\lvert T^7\rvert$	33	25
$\lvert T^8\rvert$	27	20
$\lvert T^9\rvert$	21	18
$\lvert T^{10}\rvert$	18	16
$\lvert T^{11}\rvert$	18	13
$\lvert T^{12}\rvert$		8
$\lvert T^{13}\rvert$		4
$\lvert T^{14}\rvert$		2
$\lvert T^{15}\rvert$		1
$\lvert T^{16}\rvert$		0

Table: Testing the never-covered property on one subject in the Choi-Fisman-Gale-Kariv (2007) experiment.

Application 3: Multi-rationale behavior

Let Σ be a finite collection of subsets of X (the space of alternatives).

Let $f : \Sigma \to X$ be correspondence with the property that $f(A) \subseteq A$.

Question: when is (Σ, f) multi-rationalizable, i.e.,

there a set Π of *strict* preferences such that

$$f(A) = \{x \in A : \ x \succ y \text{ for all } y \in A \text{ for some } \succ \in \Pi\}.$$

Application 3: Multi-rationale behavior

Let Σ be a finite collection of subsets of X (the space of alternatives).

Let $f : \Sigma \to X$ be correspondence with the property that $f(A) \subseteq A$.

Question: when is (Σ, f) multi-rationalizable, i.e.,

there a set Π of *strict* preferences such that

$$f(A) = \{x \in A : \ x \succ y \text{ for all } y \in A \text{ for some } \succ \in \Pi\}.$$

Example. Suppose $X = \{x, y, z\}$ and $\Sigma = \{\{x, y\}, \{y, z\}, \{x, y, z\}\}$.

$f(\{x, y\}) = x$; $f(\{y, z\}) = y$; $f(\{x, y, z\}) = \{x, z\}$.

(Σ, f) is not multi-rationalizable.

Application 3: Multi-rationale behavior

There is already an axiomatization of this model.

Theorem. (Aizerman and Malichevski (1981)

Let $\Sigma = \mathcal{X}$, the set of all nonempty subsets of of X (the space of alternatives).

Then the correspondence $f : \mathcal{X} \to X$, where $f(A) \subseteq A$ for all $A \in \mathcal{X}$ is multi-rationalizable if and only if it satisfies the following two conditions:

$A \subseteq B \Rightarrow f(B) \cap A \subseteq f(A)$ for all $A, B \in \mathcal{X}$ (Chernoff);

$f(B) \subseteq A \subseteq B \Rightarrow f(A) \subseteq f(B)$ for all $A, B \in \mathcal{X}$ (Aizerman).

Application 3: Multi-rationale behavior

There is already an axiomatization of this model.

Theorem. (Aizerman and Malichevski (1981)

Let $\Sigma = \mathcal{X}$, the set of all nonempty subsets of of X (the space of alternatives).

Then the correspondence $f : \mathcal{X} \to X$, where $f(A) \subseteq A$ for all $A \in \mathcal{X}$ is multi-rationalizable if and only if it satisfies the following two conditions:

$A \subseteq B \Rightarrow f(B) \cap A \subseteq f(A)$ for all $A, B \in \mathcal{X}$ (Chernoff);

$f(B) \subseteq A \subseteq B \Rightarrow f(A) \subseteq f(B)$ for all $A, B \in \mathcal{X}$ (Aizerman).

Our question:

How do we characterize multi-rationalizability when $\Sigma \neq \mathcal{X}$?

Application 3: Multi-rationale behavior

Suppose (Σ, f) is multi-rationalizable.

Then for a given $A \in \Sigma$ and $x \in f(A)$, there is $\succ \in \Pi$ such that x is optimal in A according to $\succ$. Thus

$$\mathcal{M}_{A,x} := \{(x, A \setminus x)\} \cup \big\{(f(A'), g(A'))\big\}_{A' \in \Sigma,\, A' \neq A}$$

must be rationalizable by a strict preference.
(All menu preference pairs are strict and $g(A') := A' \setminus f(A')$.)

Application 3: Multi-rationale behavior

Suppose (Σ, f) is multi-rationalizable.

Then for a given $A \in \Sigma$ and $x \in f(A)$, there is $\succ \in \Pi$ such that x is optimal in A according to $\succ$. Thus

$$\mathcal{M}_{A,x} := \{(x, A \setminus x)\} \cup \big\{(f(A'), g(A'))\big\}_{A' \in \Sigma,\, A' \neq A}$$

must be rationalizable by a strict preference.
(All menu preference pairs are strict and $g(A') := A' \setminus f(A')$.)

Conversely, if $\mathcal{M}_{A,x}$ is rationalizable by some strict preference $\succ_{A,x}$, then

$$\Pi = \{\succ_{A,x} \colon A \in \Sigma,\ x \in f(A)\}$$

would multi-rationalize (Σ, f).

Application 3: Multi-rationale behavior

Theorem. The following statements on (Σ, f) are equivalent.

(1) (Σ, f) is multi-rationalizable.

(2) For each $A \in \Sigma$ and $x \in f(A)$,

$$\mathcal{M}_{A,x} := \{(x, A \setminus x)\} \cup \big\{(f(A'), g(A'))\big\}_{A' \in \Sigma,\, A' \neq A}$$

is rationalizable as strict menu preference pairs.

(3) For any nonempty $\Sigma' \subseteq \Sigma$ and $B \in \Sigma$,

$$\big(\cup_{A \in \Sigma'} f(A) \setminus \cup_{A \in \Sigma'} g(A)\big) \subseteq B \Longrightarrow f(B) \cap \big(\cup_{A \in \Sigma'} g(A)\big) = \emptyset.$$

The last condition can be checked by an efficient algorithm.

Conclusion

For a data set of menu preference pairs

$$\mathcal{M} := \{(A^t, B^t)\}_{t \in T},$$

we provide an **implementable** way of testing if it is rationalizable.

The paper gives four applications of our basic result:

(1) A characterization of indirect preference (i.e preference over prices) on finite data.

(2) A generalization of Afriat's Theorem to data sets with imperfectly observed choices.

(3) A test of the multiple preferences model.

(4) A test of choice behavior generated by minimax regret.