Comparative statics with adjustment costs and the le Chatelier principle

Eddie Dekel, John K.-H. Quah, Ludvig Sinander

Basic concepts

A partially ordered set $(X, \geqslant_X)$ is a lattice if every two elements has a least upper bound (supremum) and a greatest lower bound (infimum).

Denote the supremum of x and x' by $x \vee x'$ and the infimum by $x \wedge x'$.

Example. $(\mathbb{R}^\ell, \geqslant)$ is a lattice, where $\geqslant$ is the product order, i.e. $x' \geqslant x$ if $x_i' \geqslant x_i$ for $i = 1, 2, .., \ell$. Indeed,

$$\begin{aligned} x \vee x' &= (\max\{x_1, x_1'\}, \max\{x_2, x_2'\}, ..., \max\{x_\ell, x_\ell'\}) \\ x \wedge x' &= (\min\{x_1, x_1'\}, \min\{x_2, x_2'\}, ..., \min\{x_\ell, x_\ell'\}). \end{aligned}$$

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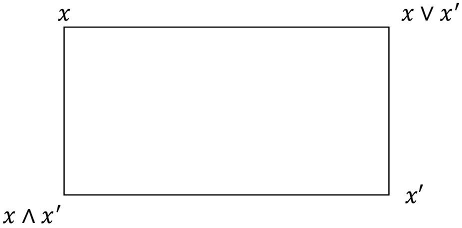

Basic concepts

Example. Distributions on $S \subset \mathbb{R}$ form a lattice when they are ordered by first order stochastic dominance.

$$\begin{aligned}(\lambda \vee \lambda')(s) &= \min\{\lambda(s), \lambda'(s)\} \\ (\lambda \wedge \lambda')(s) &= \max\{\lambda(s), \lambda'(s)\}\end{aligned}$$

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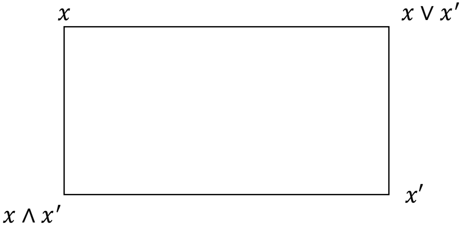
Definition. A function $F : (X, \geqslant_X) \to \mathbb{R}$ is supermodular if

$$F(x) + F(x') \leqslant F(x \wedge x') + F(x \vee x') \text{ for any } x,\ x' \in X.$$



Basic concepts

For $F : (\mathbb{R}^\ell, \geqslant) \to \mathbb{R}$, supermodularity is equivalent to the following:

for any $i \in \{1, 2, ..., \ell\}$, with $x_i'' > x_i'$,

$$F(x_i'', x_{-i}) - F(x_i', x_{-i}) \text{ is increasing in } x_{-i}.$$

Equivalently,

$$\frac{\partial F}{\partial x_i}(x_i, x_{-i}) \geqslant 0 \text{ is increasing in } x_{-i}.$$

If F is a production function, this says that the marginal productivity of factor i increases as the input of other factors, x_{-i} is raised.

If F is C^2, the supermodularity of F is equivalent to

$$\frac{\partial^2 F}{\partial x_i \partial x_j}(x) \geqslant 0 \text{ for all } x, \text{ and } i \neq j.$$

Basic concepts

Supermodularity can be equivalently re-written as

$$F(x) - F(x \wedge x') \leqslant F(x \vee x') - F(x') \text{ for any } x,\ x' \in X.$$

Definition. $F : X \to \mathbb{R}$ is a quasisupermodular function if

$$F(x) - F(x \wedge x') \geqslant 0 \implies F(x \vee x') - F(x) \geqslant 0 \text{ and}$$
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This is the ordinal version of supermodularity.

Basic concepts

Example. Suppose a firm chooses bundle with ℓ factors to maximize profit.

Formally, it chooses $x \in X$, a sublattice of $\mathbb{R}_+^\ell$, which gives revenue $R(x)$.

At factor prices $p = (p_1, p_2, \ldots, p_\ell) \gg 0$, the profit is

$$\pi(x, p) = R(x) - p \cdot x.$$

If R is supermodular (so factors are complements), then $\pi(x, p)$ is also a supermodular function of x.

Basic concepts

Example. Let I and S be subsets of $\mathbb{R}$ and let $u : I \times S \to \mathbb{R}$ be a supermodular function: if $y' > y$ and $s' \geqslant s$, then

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Suppose the state s is uncertain and let $U(y, s) = \int_S u(y, s)\, d\lambda(s)$.

Then, if $y' > y$ and $\lambda' \geqslant \lambda$

$$U(y', \lambda) - U(y, \lambda) \leqslant U(y', \lambda') - U(y, \lambda');$$

equivalently,

$$U(y', \lambda) + U(y, \lambda') \leqslant U(y, \lambda) + U(y', \lambda').$$

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More generally, U is a supermodular function, i.e.,

$$U(y, \lambda) + U(y', \lambda') \leqslant U(y \vee y', \lambda \vee \lambda') + U(y \wedge y', \lambda \wedge \lambda').$$

Basic concepts: comparing objective functions

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Let $(X, \geqslant_X)$ be a lattice and Θ a poset.

Let $\{F(\cdot, \theta)\}_{\theta \in \Theta}$ be a family of objective functions defined on X (the set of actions) and parameterized by $\theta \in \Theta$.

Definition. The family $\{F(\cdot, \theta)\}_{\theta \in \Theta}$ satisfies single crossing differences if, for $x' > x$ and $\theta' > \theta$,

$$F(x', \theta) - F(x, \theta) \geqslant (>)\, 0 \Longrightarrow F(x', \theta') - F(x, \theta') \geqslant (>)\, 0.$$

This property holds when $\{F(\cdot, \theta)\}_{\theta \in \Theta}$ satisfies increasing differences, i.e, for all $x' > x$ and $\theta' > \theta$,

$$F(x', \theta') - F(x, \theta') \geqslant F(x', \theta) - F(x, \theta).$$

Basic concepts: comparing sets

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For x' and x in $\mathbb{R}^\ell$, we know what it means for x' to be higher than x.

What does it for a *set* A' to be higher than A?

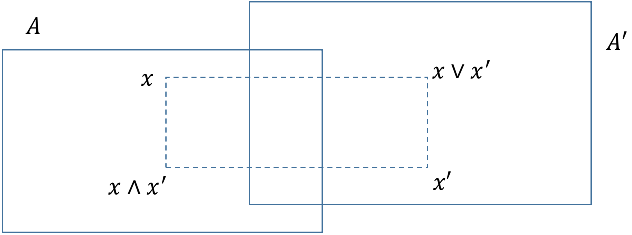
Very strong notion: every $x' \in A'$ is higher than every $x \in A$.

Very weak notion: for every $x \in A$, there is $x' \in A'$ such that $x' \geqslant x$ and vice versa.

The best notion is somewhere in between.

Basic concepts: comparing sets

A set A' dominates A by the strong set order if for all $x' \in A'$ and $x \in A$, we have $x' \vee x \in A'$ and $x' \wedge x \in A$.



This captures the idea that A' takes higher values than A.

If A' and A are singleton sets containing x' and x, then $x' \geqslant x$ if A' dominates A by the strong set order.

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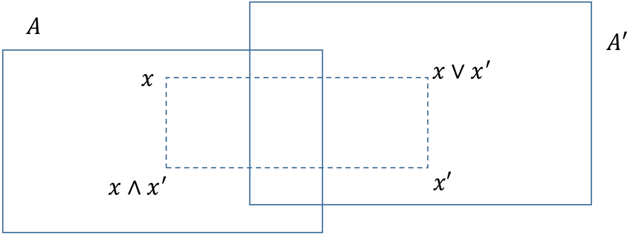

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If A' and A are singleton sets containing x' and x, then $x' \geqslant x$ if A' dominates A by the strong set order.

If A' and A are nonempty, then $A' \geqslant A$ implies the following: for all $x \in A$, there is $y \in A'$ such that $y \geqslant x$.

Why? Choose any $x' \in A'$ and let $y = x \vee x'$.

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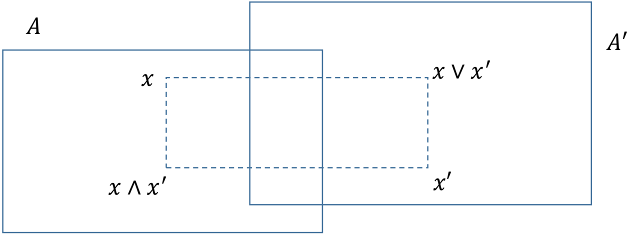

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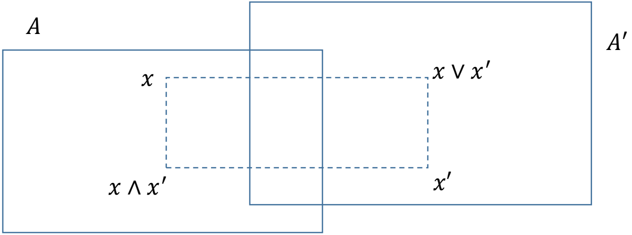

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If A' and A are compact sublattices, $\sup A' \geqslant \sup A$ and $\inf A' \geqslant \inf A$.

Monotone Comparative Statics (Milgrom-Shannon)

Theorem. Suppose $\{F(\cdot,\theta)\}_{\theta\in\Theta}$ is a family of quasisupermodular functions that satisfy single crossing differences.

Then for any $\bar{\theta} > \underline{\theta}$ and $X(\bar{\theta}) \geqslant X(\underline{\theta})$,

$$\text{argmax}_{x\in X(\bar{\theta})} F(x,\bar{\theta}) \geqslant \text{argmax}_{x\in X(\underline{\theta})} F(x,\underline{\theta}).$$

Interpretation:

Initially, $\theta = \underline{\theta}$ and the agent's action is $\underline{x} \in \text{argmax}_{x\in X(\underline{\theta})} F(x,\underline{\theta})$.

Suppose θ increases to $\bar{\theta}$.

Then there is an action $\bar{x} \in \text{argmax}_{x\in X(\bar{\theta})} F(x,\bar{\theta})$ such that $\bar{x} \geqslant \underline{x}$.

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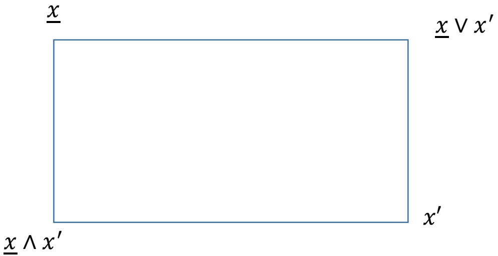
Special cases: (1) $X(\underline{\theta}) = X(\bar{\theta})$.

(2) F is quasisupermodular function of x and independent of θ. Then $\text{argmax}_{x\in X(\bar{\theta})} F(x) \geqslant \text{argmax}_{x\in X(\underline{\theta})} F(x)$ if $X(\bar{\theta}) \geqslant X(\underline{\theta})$.

MCS Theorem



Proof. Let $\underline{x} \in \text{argmax}_{x \in X(\underline{\theta})} F(x, \underline{\theta})$ and $x' \in \text{argmax}_{x \in X(\bar{\theta})} F(x, \bar{\theta})$.

Since $X(\bar{\theta}) \geqslant X(\underline{\theta})$, we have $\underline{x} \vee x' \in X(\bar{\theta})$ and $\underline{x} \wedge x' \in X(\underline{\theta})$.

We need to show that $\underline{x} \vee x' \in \text{argmax}_{x \in X(\bar{\theta})} F(x, \bar{\theta})$.

By the optimality of $\underline{x}$, we have $F(\underline{x}, \underline{\theta}) \geqslant F(\underline{x} \wedge x', \underline{\theta})$.

By quasisupermodularity, we obtain $F(\underline{x} \vee x', \underline{\theta}) \geqslant F(x', \underline{\theta})$.

By single crossing differences, $F(\underline{x} \vee x', \bar{\theta}) \geqslant F(x', \bar{\theta})$.

Thus $\underline{x} \vee x' \in \text{argmax}_{x \in X} F(x, \bar{\theta})$.

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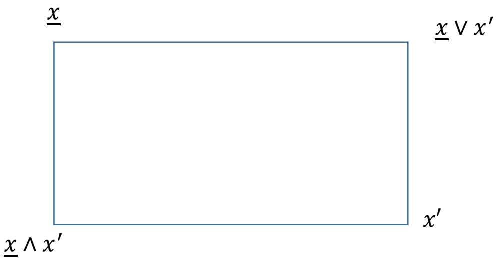

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Similarly, we can show that $\underline{x} \wedge x' \in \text{argmax}_{x \in X} F(x, \underline{\theta})$. QED

MCS Theorem: Application

Suppose a firm chooses a factor bundle $x \in X$ a sublattice of $\mathbb{R}^{\ell}_{+}$ to maximize profit. At factor prices $p = (p_1, p_2, \ldots, p_\ell) \gg 0$, profit is

$$\pi(x, p) = R(x) - p \cdot x.$$

If R is supermodular, then $\pi(x, p)$ is also a supermodular function of x.
It also has single crossing differences in $(x, -p)$, i.e.,
for all $x' > x$ and $-p' > -p$,

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By the Milgrom-Shannon Theorem,

$$\text{argmax}_{x \in X} \pi(x, p') \geqslant \text{argmax}_{x \in X} \pi(x, p) \ \text{ if } p' < p.$$

In particular suppose $p'_i = p_i$ for all $i > 2$ and $p'_1 < p_1$.
Then the demand for *all* factors increase as the price of factor 1 falls.

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Initially, $\theta = \underline{\theta}$ and the agent's action is $\underline{x} \in \text{argmax}_{x\in X(\underline{\theta})} F(x,\underline{\theta})$.

Suppose θ increases to $\bar{\theta}$.

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Adjustment costs

Changing the action from the initial choice of $\underline{x}$ is costly.

Example. Suppose there is a cost associated with the adjustment of each factor i. If r is the adjustment, then the cost is $C_i(r)$.

Total adjustment cost from $\underline{x}$ to another choice x is then

$$C(x, \underline{x}) = \sum_{i=1}^{\ell} C_i(x_i - \underline{x}_i).$$

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It is natural to assume that C_i is U-shaped with a minimum at 0.

Under this assumption, C has the monotone property.

Definition. $C : X \times X \to \mathbb{R}$ has the monotone property if

$$C(x', \underline{x}) \geqslant C(x, \underline{x})$$

whenever $x_i = x_i'$ for all $i \neq k$ and either $x_k' \geqslant x_k \geqslant \underline{x}_k$ or $x_k' \leqslant x_k \leqslant \underline{x}_k$.

Adjustment costs

For example, monotonicity requires

$C((4,1,3),(2,2,2)) \geqslant C((3,1,3),(2,2,2))$,
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Definition. $C : X \times X \to \mathbb{R}$ has the minimally monotone property if

$C(x,\underline{x}) \geqslant C(x \vee \underline{x}, \underline{x})$ and $C(x,\underline{x}) \geqslant C(x \wedge \underline{x}, \underline{x})$.

If C is monotone then it is minimally monotone.

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$C((4,1,3),(2,2,2)) \geqslant C((2,1,2),(2,2,2))$ and
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Both the monotone and minimal monotone properties are *ordinal*.

MCS Theorem with Adjustment Costs

Theorem. Suppose $\{F(\cdot,\theta)\}_{\theta\in\Theta}$ is a family of quasisupermodular functions that satisfy single crossing differences and suppose that the adjustment cost function C is minimally monotone.

For $\bar{\theta} > \underline{\theta}$ and $X(\bar{\theta}) \geqslant X(\underline{\theta})$, let $\underline{x} \in \text{argmax}_{x\in X(\underline{\theta})} F(x,\underline{\theta})$ and $x' \in \text{argmax}_{x\in X(\bar{\theta})} \left\{F(x,\bar{\theta}) - C(x,\underline{x})\right\}$.

Then $x' \vee \underline{x} \in \text{argmax}_{x\in X(\bar{\theta})} \left\{F(x,\bar{\theta}) - C(x,\underline{x})\right\}$.

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Then $x' \vee \underline{x} \in \text{argmax}_{x\in X(\bar{\theta})} \left\{F(x,\bar{\theta}) - C(x,\underline{x})\right\}$.

Adjustment costs do not alter the *direction* of responses:

given $\underline{x} \in \text{argmax}_{x\in X(\underline{\theta})} F(x,\underline{\theta})$, there is $\hat{x} \in \text{argmax}_{x\in X(\bar{\theta})} \left\{F(x,\bar{\theta}) - C(x,\underline{x})\right\}$ such that $\hat{x} \geqslant \underline{x}$.

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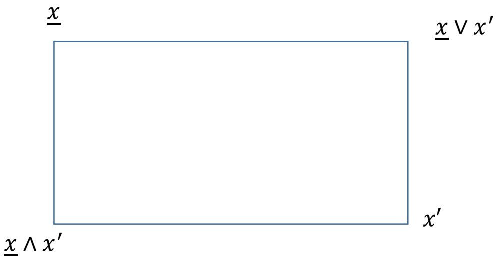

Proof.

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Since $X(\bar{\theta}) \geqslant X(\underline{\theta})$, we have $\underline{x} \vee x' \in X(\bar{\theta})$ and $\underline{x} \wedge x' \in X(\underline{\theta})$.

By the optimality of $\underline{x}$, we have $F(\underline{x}, \underline{\theta}) \geqslant F(\underline{x} \wedge x', \theta)$.

Quasisupermodularity then guarantees that $F(\underline{x} \vee x', \underline{\theta}) \geqslant F(x', \underline{\theta})$.

By single crossing differences, $F(\underline{x} \vee x', \bar{\theta}) \geqslant F(x', \bar{\theta})$.

Since C is minimally monotone, $C(x', \underline{x}) \geqslant C(x' \vee \underline{x}, \underline{x})$.

Thus $F(x' \vee \underline{x}, \bar{\theta}) - C(x' \vee \underline{x}, \underline{x}) \geqslant F(x', \bar{\theta}) - C(x', \underline{x})$.

We conclude that $x' \vee \underline{x} \in \text{argmax}_{x \in X} F(x, \bar{\theta}) - C(x, \underline{x})$ ... QED

Application: costly factor adjustments

A firm chooses a factor bundle $x \in X$, a sublattice of $(\mathbb{R}^\ell_+, \geqslant)$, to maximize profit $\pi(x, p) = R(x) - p \cdot x$, where

$p = (p_1, p_2, \ldots, p_\ell) \gg 0$ are the factor prices.

If R is supermodular, then $\pi(x, p)$ is a supermodular function of x and has single crossing differences in (x, θ) where $\theta = -p$.

The initial price vector is $\underline{p}$, with the firm choosing $\underline{x} \in \text{argmax}_{x \in X} \pi(x, \underline{p})$.

Suppose prices drop to $\bar{p} < \underline{p}$. By MCS theorem, there is

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If adjustment cost C is minimally monotone, our theorem guarantees that there is

$\hat{x} \in \text{argmax}_{x \in X} \{\pi(x, p) - C(x, \underline{x})\}$ such that $\underline{x} \leqslant \hat{x}$.

Application: wishful thinking

Agent has income w in period 1 and uncertain income s in period 2.

The agent chooses saving y in some interval I of $\mathbb{R}$ to maximize discounted expected utility

$$U(y, \lambda) = \int_S [u_1(w - y) + u_2(Ry + s)] \; d\lambda(s).$$

If u_2 is concave, the map $(-y, s) \to u_1(w - y) + u_2(Rx + s)$ is supermodular.

Thus the map $(-y, \lambda) \to U(y, \lambda)$ is also supermodular.

In particular, for $y' > y$ and $\lambda' >_{FSD} \lambda$,

$$U(y', \lambda) - U(y, \lambda) \geqslant U(y', \lambda') - U(y, \lambda').$$

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By MCS theorem, if $\lambda' >_{FSD} \lambda$, then

$$\text{argmax}_{y \in I} U(y, \lambda') \leqslant \text{argmax}_{y \in I} U(y, \lambda).$$

Application: wishful thinking

Suppose that the true distribution is $\underline{\lambda}$, so the agent who can handle the truth chooses

$$\underline{y} \in \mathrm{argmax}_{y \in I} U(y, \underline{\lambda}).$$

Consider a wishful thinking agent of the Caplin-Leahy type.

Agent chooses λ from an interval $\Lambda = [\lambda_0, \bar{\lambda}]$ (which we assume contains $\underline{\lambda}$) and $y \in I$ to maximize

$$F(y, \lambda) = U(y, \lambda) - C(\lambda, \underline{\lambda}),$$

where C is the cost of deviating from the true distribution $\underline{\lambda}$.

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where C is the cost of deviating from the true distribution $\underline{\lambda}$.

If $C \equiv 0$, then the agent will simply choose $\lambda = \bar{\lambda}$ (the most optimistic scenario) and saving $\bar{y} \in \text{argmax}_{y \in I} U(y, \bar{\lambda})$.

By MCS Theorem, $(-\underline{y}, \underline{\lambda}) \leqslant (-\bar{y}, \bar{\lambda})$. But how does

$(\hat{y}, \hat{\lambda}) \in \text{argmax}_{(y,\lambda) \in I \times \Lambda} F(y, \lambda)$ compare with $(\underline{y}, \underline{\lambda})$ and $(\bar{y}, \bar{\lambda})$?

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Claim. there is $(\hat{y}, \hat{\lambda}) \in \operatorname{argmax}_{(y,\lambda)\in I\times\Lambda} F(y,\lambda)$ such that

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Proof. Since $\hat{\lambda} \in \Lambda = [\lambda_0, \bar{\lambda}]$, we have $\hat{\lambda} \leqslant \bar{\lambda}$ and hence $-\hat{y} \leqslant -\bar{y}$.

It remains to show that $(-\underline{y}, \underline{\lambda}) \leqslant (-\hat{y}, \hat{\lambda})$.

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Claim. there is $(\hat{y}, \hat{\lambda}) \in \text{argmax}_{(y,\lambda)\in I\times\Lambda} F(y, \lambda)$ such that

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It remains to show that $(-\underline{y}, \underline{\lambda}) \leqslant (-\hat{y}, \hat{\lambda})$.

Let $\underline{\Lambda} = [\lambda_0, \underline{\lambda}]$. Re-write the rational agent's problem as one to

maximize $U(y, \lambda)$ *subject to* $(y, \lambda) \in I \times \underline{\Lambda}$.

This gives $(\underline{y}, \underline{\lambda})$ as a solution.

On the other hand, the wishful thinking agent's problem is to

maximize $U(y, \lambda) - C(\lambda, \underline{\lambda})$ *subject to* $(y, \lambda) \in I \times \Lambda$.

Note that $\underline{\Lambda} \leqslant \Lambda$ (in the strong set order).

If C is minimally monotone, our theorem applies and gives $(-\underline{y}, \underline{\lambda}) \leqslant (-\hat{y}, \hat{\lambda})$. QED

Application: wishful thinking

Example. if C has the relative entropy form

$$C(\lambda, \underline{\lambda}) = \sum_{s \in S} \lambda(s) \ln \left(\frac{\lambda(s)}{\underline{\lambda}(s)} \right)$$

then C is minimally monotone, i.e., for any λ,

$$C(\lambda, \underline{\lambda}) \geqslant C(\lambda \vee \underline{\lambda}, \underline{\lambda}) \text{ and } C(\lambda, \underline{\lambda}) \geqslant C(\lambda \wedge \underline{\lambda}, \underline{\lambda}).$$

Le Chatelier Principle

Theorem. Suppose $\{F(\cdot, \theta)\}_{\theta \in \Theta}$ is a family of quasisupermodular functions that satisfy single crossing differences and suppose that the adjustment cost function C is monotone.

Let $\underline{\theta} \leqslant \bar{\theta}$ and $X(\underline{\theta}) \leqslant X(\bar{\theta})$, with $\underline{x} \leqslant \bar{x}$, where

$\underline{x} \in \text{argmax}_{x \in X(\underline{\theta})} F(x, \underline{\theta})$ and $\bar{x} \in \text{argmax}_{x \in X(\bar{\theta})} F(x, \bar{\theta})$.

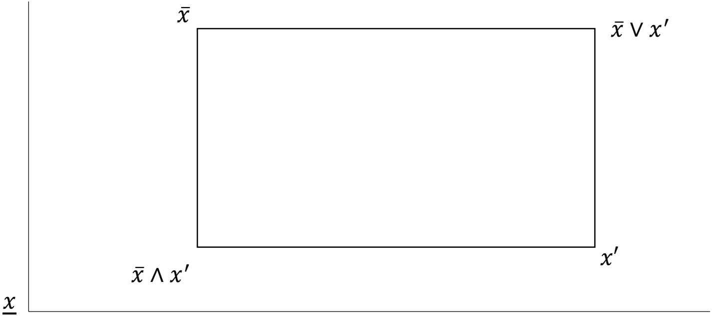
Then there is $\hat{x} \in \text{argmax}_{x \in X(\bar{\theta})} \left\{ F(x, \bar{\theta}) - C(x, \underline{x}) \right\}$ such that

$$\underline{x} \leqslant \hat{x} \leqslant \bar{x}.$$

Interpretation:

the short run adjustment is in the same direction as the long run adjustment, but smaller.

Le Chatelier Principle



Proof. From previous result we know there is $x' \in \text{argmax}_{x \in X(\bar{\theta})} F(x, \bar{\theta}) - C(x, \underline{x})$ such that $\underline{x} \leqslant x'$. Let $\hat{x} = x' \wedge \bar{x}$. Then $\underline{x} \leqslant \hat{x} \leqslant \bar{x}$.

Note that $C(\hat{x}, \underline{x}) \leqslant C(x', \underline{x})$, so if $\hat{x} \notin \text{argmax}_{x \in X(\bar{\theta})} F(x, \bar{\theta}) - C(x, \underline{x})$, then $F(\hat{x}, \bar{\theta}) < F(x', \bar{\theta})$. By quasisupermodularity of $F(\cdot, \bar{\theta})$ we obtain $F(\bar{x}, \bar{\theta}) < F(x' \vee \bar{x}, \bar{\theta})$, contradicting the optimality of $\bar{x}$. QED

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In earlier formulations of the Le Chatelier principle (Samuelson (1947), Mligrom-Roberts (1996)), some inputs (for example, capital) are held fixed in the short run.

There are fixed and variable inputs, so $x = (x_f, x_v)$.

Theorem. Suppose $\{F(\cdot, \theta)\}_{\theta \in \Theta}$ is a family of quasisupermodular functions that satisfy single crossing differences.

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Then there is $\hat{x}_v \in \text{argmax}_{x_v \in X_v} F(\underline{x}_f, x_v, \bar{\theta})\}$ such that $\underline{x}_v \leqslant \hat{x}_v \leqslant \bar{x}_v$.

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Special case of our result where $C(x, \underline{x}) = \sum_{i=1}^{\ell} C_i(x_i - \underline{x}_i)$

$C_i(r) = 0$ for all r if i is a variable input and $C_i(r) = \infty$ for all $r \neq 0$ if i is a fixed input.

Le Chatelier Principle: application

Suppose a firm chooses a factor bundle $x \in X$, a sublattice of $\mathbb{R}^{\ell}_{+}$, to maximize profit

$$\pi(x, p) = R(x) - p \cdot x,$$

where $p = (p_1, p_2, \ldots, p_\ell) \gg 0$ are the factor prices.

If R is supermodular, then $\pi(x, p)$ is a supermodular function of x and has single crossing differences in (x, θ) where $\theta = -p$.

Let $\underline{x} \in \text{argmax}_{x \in X} \pi(x, \underline{p})$.

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We interpret $\bar{x}$ as the firm's long-run response.

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We interpret $\bar{x}$ as the firm's long-run response.

In the short-run, the firm chooses $\hat{x} \in X$ to maximize

$$\pi(x, \bar{p}) - C(x, \underline{x}).$$

By our le Chatelier principle, there is $\hat{x}$ such that $\underline{x} \leqslant \hat{x} \leqslant \bar{x}$.

Dynamic Le Chatelier Principle

In each period, the agent takes an action $x \in X$, and earns a payoff of $F(x, \theta)$.

There is a one-time permanent shift of the parameter from $\underline{\theta}$ to $\bar{\theta} > \underline{\theta}$.

If $F(x, \theta)$ is quasisupermodular in x and has single crossing differences in (x, θ), then there is

$$\underline{x} \in \arg\max_{x \in X} F(x, \underline{\theta}) \quad \text{and} \quad \bar{x} \in \arg\max_{x \in X} F(x, \bar{\theta}).$$

such that $\underline{x} \leqslant \bar{x}$.

Absent adjustment costs, the agent will switch immediately from $\underline{x}$ to $\bar{x}$.

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Absent adjustment costs, the agent will switch immediately from $\underline{x}$ to $\bar{x}$.

What is the trajectory of agent actions if there are adjustment costs?

Dynamic Le Chatelier Principle

The forward-looking decision maker faces an infinite-horizon decision problem in discrete time: chooses $\{x^t\}_{t=1}^{\infty}$ to maximize

$$\sum_{t=1}^{\infty} \delta^{t-1} \left[F(x^t, \bar{\theta}) - C^t(x^t, x^{t-1} \right]$$

with initial condition $x^0 = \underline{x}$.

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Theorem. Suppose $\{F(\cdot, \theta)\}_{\theta \in \Theta}$ is a family of quasisupermodular functions that satisfy single crossing differences and suppose that the adjustment cost function in period t, C^t is monotone.

Then there is a solution $\{\hat{x}^t\}_{t=1}^{\infty}$ to the infinite horizon optimization problem with adjustment costs such that

$$\underline{x} \leqslant \hat{x}^t \leqslant \bar{x} \;\text{ for all } t.$$

Dynamic Le Chatelier Principle

The adjustment cost function is additively separable if $C(x, y) = \sum_{i=1}^{\ell} C_i(x_i, y_i)$ for all i.

C is monotone if and only if C_i is U-shaped function of x_i, with a minimum at y_i.

C is convex in x if and only if C_i is convex in x_i for all i.

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C is monotone if and only if C_i is U-shaped function of x_i, with a minimum at y_i.

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Theorem. Suppose $\{F(\cdot, \theta)\}_{\theta \in \Theta}$ is a family of supermodular functions that satisfy single crossing differences and suppose that the adjustment cost function in period t, $C^t \equiv C$ for all t, with C being additively separable and monotone.

Then there is a solution $\{\hat{x}^t\}_{t=1}^{\infty}$ to the infinite horizon optimization problem with adjustment costs such that

$$\underline{x} \leqslant \hat{x}^1 \leqslant \hat{x}^2 \leqslant \hat{x}^3 \leqslant \ldots \leqslant \bar{x}.$$

Dynamic Le Chatelier Principle: application

A monopolist chooses price p to maximize profit

$$F(p,(c,\eta)) = (p-c)D(p,\eta).$$

This is trivially supermodular in p.

$\ln F = \ln(p-c) + \ln D(p,\eta)$ has increasing differences in (p,c).

$\ln F$ also has increasing differences in (p,η) provided the price elasticity of demand

$$-\frac{p}{D(p,\eta)}\frac{\partial D}{\partial p}(p,\eta) \text{ is decreasing in } \eta.$$

Thus F has single crossing differences in $(p,(c,\eta))$ if we assume that the elasticity of demand falls with η.

Dynamic Le Chatelier Principle: application

Assuming that the elasticity of demand is falling with η,

$$F(p,(c,\eta)) = (p-c)D(p,\eta)$$

has single crossing differences in $(p,(c,\eta))$. It is also trivially supermodular in p.

The MCS theorem applies. With an upward shift from $(\underline{c},\underline{\eta})$ to $(\bar{c},\bar{\eta})$,

there is $\underline{p} \in \text{argmax}_{p \in P} F(p,(\underline{c},\underline{\eta}))$ and $\bar{p} \in \text{argmax}_{p \in P} F(p,(\bar{c},\bar{\eta}))$

such that $\underline{p} \leqslant \bar{p}$.

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Suppose there is a monotone adjustment cost C. (It is trivially additively separable.)

The dynamic le Chatelier principle guarantees that

$$\underline{p} = p^0 \leqslant p^1 \leqslant p^2 \leqslant \ldots \leqslant \bar{p}.$$

Dynamic Le Chatelier Principle: application

Membership in a club costs m dollars per month.

The net benefit of spending $x > 0$ hours in the club per month is (in dollar terms) $F(x, m) = U(x) - m$.

If the potential member joins the club, then he chooses

$$\underline{x} \in \text{argmax}_{x>0} U(x).$$

Suppose the initial membership fee is $\underline{m}$ and $U(\underline{x}) - \underline{m} > 0$.

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Suppose the initial membership fee is $\underline{m}$ and $U(\underline{x}) - \underline{m} > 0$.

Membership fee goes up from $\underline{m}$ to $\overline{m}$ such that $U(\underline{x}) - \overline{m} < 0$.

Without adjustment cost/inertia, member drops out immediately. Formally, switches from $x = \underline{x}$ to $x = 0$.

With monotone adjustment cost, the le Chatelier principle says

$$\underline{x} \geqslant x^1 \geqslant x^2 \geqslant x^3 \geqslant \ldots \geqslant 0.$$

Then either $\underline{x} = x_i$ for all i or there is N such that $x_N = 0$.

Dynamic Le Chatelier Principle

We first consider a myopic decision-maker who takes into account adjustment cost but only maximizes payoff period-by-period.

If $\tilde{x}^t$ is the action in period t and $\tilde{x}^{t-1}$ the action in period $t-1$, the adjustment cost is $C^t(\tilde{x}^t, \tilde{x}^{t-1})$.

Then $\tilde{x}^t \in \text{argmax}_{x \in X} \left\{ F(x, \bar{\theta}) - C^t(\tilde{x}, \tilde{x}^{t-1}) \right\}$.

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Then there is a solution $\tilde{x}^1$, $\tilde{x}^2, \ldots$ such that

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Under further assumptions, there is a solution $\hat{x}^1$, $\hat{x}^2, \ldots$ of the forward-looking agent such that $\tilde{x}^t \leqslant \hat{x}^t$ for all t.

Conclusion

The basic results of monotone comparative statics are preserved even if there are adjustment costs.

Adjustment costs diminish the scale of adjustment but not its direction.

Adjustment costs lead to actions being adjusted over time, and the trajectory of actions can be analyzed under weak assumptions.