Intertemporal consumption with risk: a revealed preference analysis

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Introduction

We analyze choice behavior under **risk and time** in a budgetary environment.

The experiment is similar to other experiments involving budgetary choices, for example,

- risk preference (Choi, Fisman, Gale, and Kariv, 2007)
- ambiguity preference (Ahn et al., 2014)
- time preference (Andreoni and Sprenger, 2012)
- social preference (Andreoni and Miller, 2002; Fisman, Kariv, and Markovits, 2007)

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Surprisingly few experiments that involve both time and risk.

Methodological contribution: we use

new revealed preference techniques.

The experiment

There are two states which occur with equal probability.

Outcome at each state is a **consumption stream**, with a payout at date 1 (one week later) and another at date 2 (nine weeks later).

	t_1	t_2
s_1	x_{11}	x_{12}
s_2	x_{21}	x_{22}

Subjects allocate 100 tokens across the four contingent commodities. They choose $x = ((x_{11}, x_{12}), (x_{21}, x_{22}))$ to satisfy budget

$$p_{11}x_{11} + p_{12}x_{12} + p_{21}x_{21} + p_{22}x_{22} = 100.$$

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$$p_{11}x_{11} + p_{12}x_{12} + p_{21}x_{21} + p_{22}x_{22} = 100.$$

For example, if $p_{11} = 1$, $p_{12} = 1$, $p_{21} = 2$, and $p_{22} = 1$ then the bundle $x = (50, 0, 10, 30)$ is feasible since

$$1(50) + 1(0) + 2(10) + 1(30) = 100.$$

If state 2 is realized, the subject receives $10(0.2) =$SGD2 at date 1 and $30(0.2) =$SGD6 at date 2.

The experiment

- One of the four prices is always 1, the other three prices are randomly chosen from {0.5, 0.8, 1.25, 2}.
- In addition to (1, 1, 1, 1), 40 budget sets are randomly chosen for subjects in each session.
- Each subject is paid according to one decision task, chosen via the Random Incentive Mechanism.
- Subjects were paid on average SGD 22 with post-dated cheques.
- A total of 103 undergraduate students from the National University of Singapore.
- Most of our subjects completed the tasks within 40 minutes.

Discounted expected utility

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How should subject evaluate different bundles $x = (x_{11}, x_{12}, x_{21}, x_{22})$ on the budget line

$$p_{11}x_{11} + p_{12}x_{12} + p_{21}x_{21} + p_{22}x_{22} = 100\,?$$

The canonical model is discounted expected utility (DEU):

$$U(x_{11}, x_{12}, x_{21}, x_{22}) = 0.5\left[u(x_{11}) + \delta u(x_{12})\right] + 0.5\left[u(x_{21}) + \delta u(x_{22})\right].$$

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We analyze the data to answer the following questions:

- is the subject is maximizing *some* utility function

$$U(x_{11}, x_{12}, x_{21}, x_{22})?$$

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We analyze the data to answer the following questions:

- is the subject is maximizing *some* utility function

$$U(x_{11}, x_{12}, x_{21}, x_{22})?$$

- what properties does that utility function satisfy?

Revealed Preference Analysis

Let $\mathcal{O} = \{(p^t, x^t)\}_{t \in \mathcal{T}}$ be a set of observations drawn from a subject.

Each observation consists of

price vector $p^n = (p_1^n, p_2^n, \ldots, p_\ell^n) \gg 0$ and

consumption bundle $x^n = (x_1^n, x_2^n, \ldots, x_\ell^n) \geq 0$.

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Definition. A utility function $U : \mathbb{R}_+^\ell \to \mathbb{R}$ is a strictly increasing and continuous function. U rationalizes $\mathcal{O} = \{(p^n, x^n)\}_{n \in \mathcal{N}}$ if, at every observation $n \in \mathcal{N}$,

$$U(x^n) \geq U(x) \text{ for all } x \in \{x \in \mathbb{R}_+^\ell : p^n \cdot x \leq p^n \cdot x^n\}.$$

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Afriat's Theorem (1967) answers the following question:

what conditions on $\mathcal{O} = \{(p^n, x^n)\}_{n\in\mathcal{N}}$ are necessary and sufficient for it to be rationalizable by a utility function?

GARP

Given $\mathcal{O} = \{(p^n, x^n)\}_{n\in\mathcal{N}}$, let $\mathcal{D} = \{x^n\}_{n\in\mathcal{N}}$.

For any x^n, $x^m \in \mathcal{D}$, we say x^n is directly revealed preferred to x^m if $p^n \cdot x^m \leq p^n \cdot x^n$. [Notation: $x^n \succeq x^m$.]

If $p^n \cdot x^m < p^n \cdot x^n$, we say x^n is directly revealed strictly preferred to x^m. [Notation: $x^m \ggcurly x^m$.]

Motivation: For an agent maximizing a utility function U,

$$x^n \succeq x^m \implies U(x^n) \geq U(x^m) \text{ and}$$

$$x^n \ggcurly x^n \implies U(x^n) > U(x^m).$$

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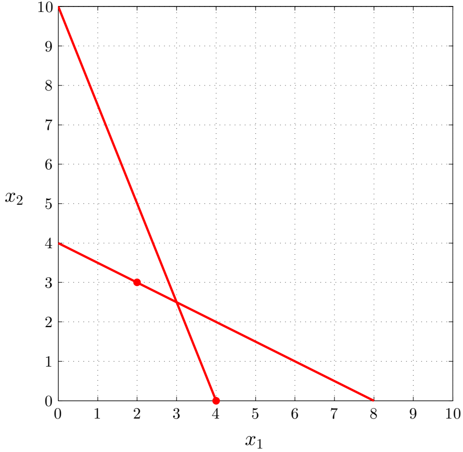
Definition. A data set $\mathcal{O} = \{(p^n, x^n)\}_{n \in \mathcal{N}}$ obeys the Generalized Axiom of Revealed Preference (GARP) if, for all sequences $n_1, \ldots, n_K$ in $\{1, 2, \ldots, N\}$,

$$x^{n_1} \succeq x^{n_2} \succeq \ldots \succeq x^{n_K} \Longrightarrow x^{n_K} \not\ggcurly x^{n_1}.$$

Afriat's Theorem. $\mathcal{O}$ can be rationalized by a utility function if and only if it obeys GARP.

GARP

Example of GARP violation:



Violation of GARP: $x^1 \ggg x^2$ and $x^2 \ggg x^1$.

Critical Cost Efficiency Index

How do we measure the extent of the departure from rationality?

We use an approach suggested by Afriat (1972) and Varian (1990).

If no increasing utility function rationalizes $\mathcal{O}$, we make the requirement *less stringent* by shrinking all budget sets in $\mathcal{O}$ by a factor $e \in [0, 1)$.

Is there U such that $U(x^t) \geq U(x)$ for all $x \in \mathcal{B}^t(e)$, where

$$\mathcal{B}^t(e) = \{x \in \mathbb{R}^S_+ : x \leq x^t\} \cup \{x \in \mathbb{R}^S_+ : p^t \cdot x \leq e\, p^t \cdot x^t\}.$$

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The largest e at which a data set passes the test is known as the critical cost efficiency index (CCEI).

e can be obtained via a modification of the GARP test.

Critical Cost Efficiency Index

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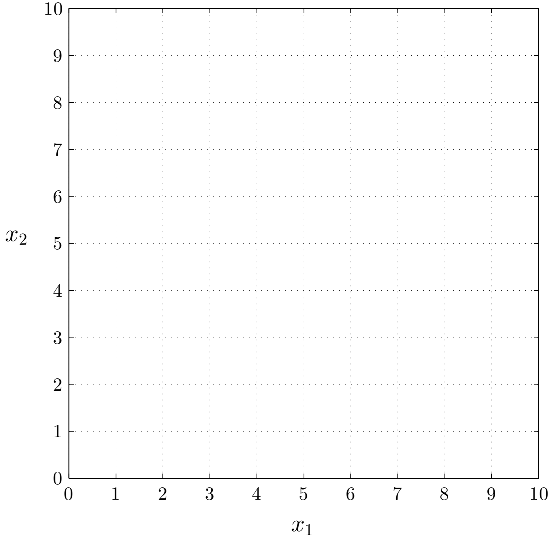

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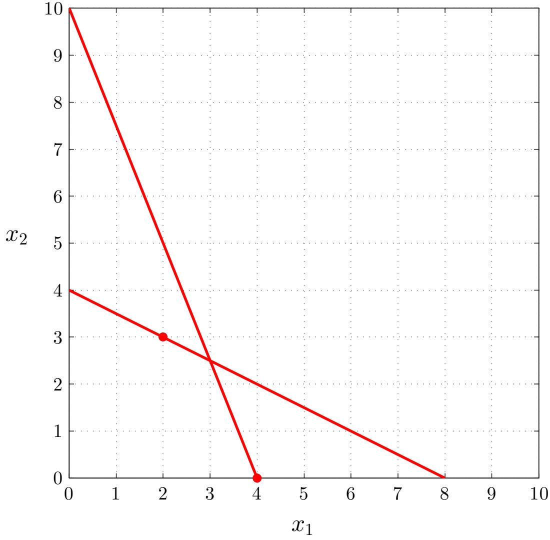

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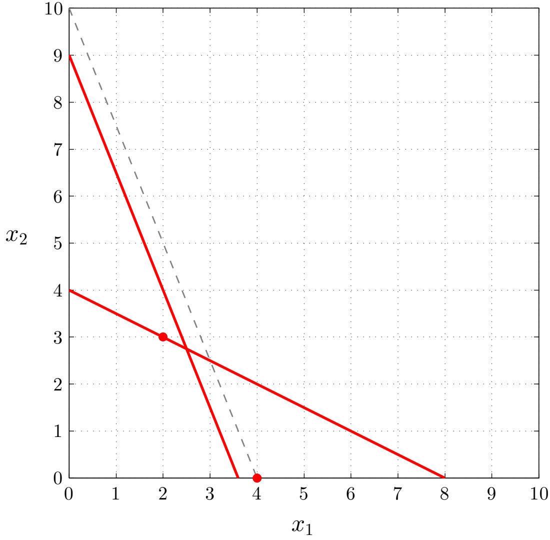

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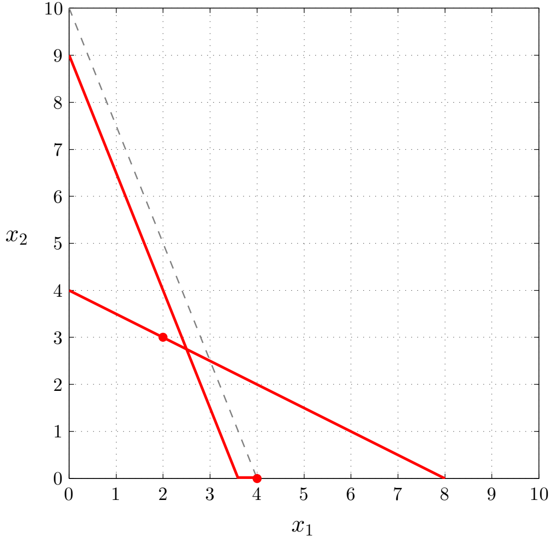

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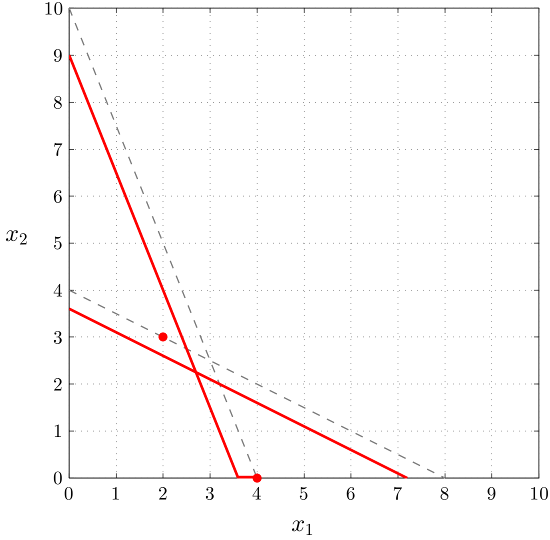

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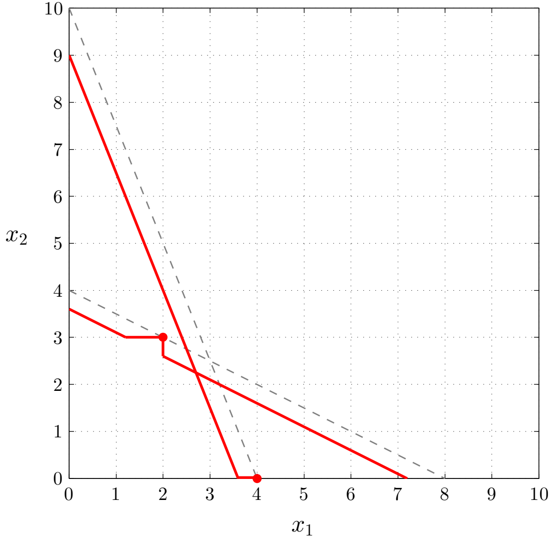

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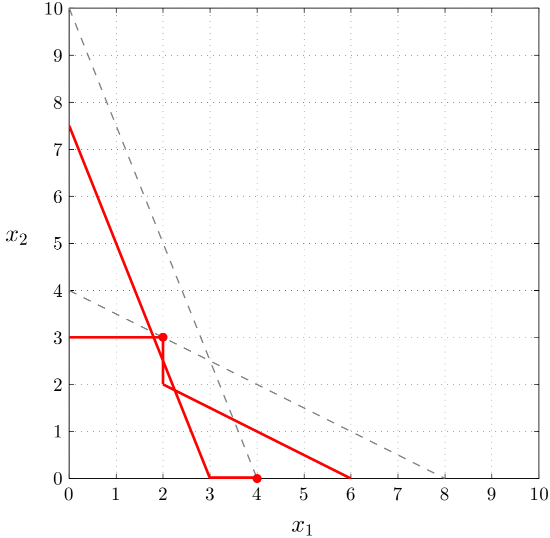

Basic Rationality

	t_1	t_2
s_1	x_{11}	x_{12}
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The data set in our case is

$$\mathcal{O} = \{[(p^n_{11}, p^n_{12}, p^n_{21}, p^n_{22})\,;(x^n_{11}, x^n_{12}, x^n_{21}, x^n_{22})]\}^{41}_{n=1}$$

We can check for utility-maximization by checking GARP.

Subjects are broadly consistent with utility-maximization.

Table: Pass Rates for Utility Maximization

	$\bar{e} \geq 0.99$	$\bar{e} \geq 0.95$	$\bar{e} \geq 0.90$
Experimental subjects	71.8	90.3	97.1
Uniform random datasets	0.0	0.0	0.0
Resampled datasets	24.3	38.6	65.7

SuperGARP

We implement tests of the rationalizability of $\mathcal{O} = \{(p^n, x^n)\}_{n \in \mathcal{N}}$ with a utility function $U : \mathbb{R}^\ell_+ \to \mathbb{R}$ that has added properties.

This is based on Nishimura, Ok, and Quah (2017).

The added property must take the form of agreement with a given underlying preorder $\trianglerighteq$ on $\mathbb{R}^\ell_+$, i.e.,

$U(x') \geq U(x)$ whenever $x' \trianglerighteq x$.

Some of the restrictions implied by DEU

$$U(x_{11}, x_{12}, x_{21}, x_{22}) = 0.5 \left[u(x_{11}) + \delta u(x_{12})\right] + 0.5 \left[u(x_{21}) + \delta u(x_{22})\right]$$

take the form of agreement with various preorders.

SuperGARP

A utility function $U : \mathbb{R}^4_+ \to \mathbb{R}$ satisfies lottery equivalence (LE) if

$$U((a,b),(a',b')) = U((a',b'),(a,b))$$

for all (a,b) and (a',b') in $\mathbb{R}^2_+$.

LE is obviously satisfied by DEU.

It also holds for any state-separable form

$$U((a,b),(a',b')) = G(f(a,b), f(a',b'))$$

with a symmetric function G.

But state-separability is not essential. For example

$$U((a,b),(a',b')) = f(a,b) + h(a,b)h(a',b') + f(a',b')$$

satisfies LE.

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LE can be re-stated as agreement with the LE preorder $\trianglerighteq_{\text{LE}}$:

$$((a,b),(a',b')) \trianglerighteq_{\text{LE}} ((a',b'),(a,b))$$

for all (a,b) and (a',b') in $\mathbb{R}^2_+$.

Is $\mathcal{O}$ rationalizable by a utility function that satisfies LE;

i.e., by a utility function that extends $\trianglerighteq = \trianglerighteq_{\text{LE}}$.

SuperGARP

Given data set $\mathcal{O} = \{(p^n, x^n)\}_{n=1}^N$,

bundle x^n is revealed preferred to x^m according to the preorder $\trianglerighteq$ if there exists bundle x such that

$$p^n \cdot x^n \geq p^n \cdot x \quad \text{and} \quad x \trianglerighteq x^m \tag{1}$$

When this occurs we write $x^n \succeq x^m$.

Bundle x^n is strictly revealed preferred to x^m according to $\trianglerighteq$

if it is possible to replace *either* $\trianglerighteq$ with $\triangleright$ *or* $\geq$ with $>$ in (1). We denote this relation by $x^n \succcurlyeq\!\!\succ x^m$.

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We denote this relation by $x^n \ggcurly x^m$.

Example. $p^n = (1, 1, 2, 2)$, $x^n = (50, 50, 0, 0)$, and $x^m = (0, 0, 40, 40)$.

Then $p^n \cdot x^m = 160 > p^n \cdot x^n = 100$, so $x^n \not\succeq x^m$.

However, $x^n \ggcurly_{\text{LE}} x^m$ since $p^n \cdot x^n > p^n \cdot (40, 40, 0, 0)$ and $(40, 40, 0, 0) \trianglerighteq_{\text{LE}} (0, 0, 40, 40)$.

SuperGARP

Definition. A dataset $\mathcal{O}$ satisfies GARP according to $\trianglerighteq$ if, for all sequences $n_1, \ldots, n_K$ in $\{1, 2, \ldots, N\}$,

$$x^{n_1} \succeq x^{n_2} \succeq \ldots \succeq x^{n_K} \Longrightarrow x^{n_K} \not\succ x^{n_1}.$$

Theorem. Suppose $\trianglerighteq$ is a well-behaved and closed preorder.

Then $\mathcal{O}$ is rationalizable by a utility function that agrees with $\trianglerighteq$ if and only if it obeys GARP according to $\trianglerighteq$.

We can also calculate Afriat's efficiency index for this model, i.e., the largest e such that there is a utility function U that agrees with $\trianglerighteq$ and satisfies $U(x^t) \geq U(x)$ for all $x \in \mathcal{B}^t(e)$,

$$\mathcal{B}^t(e) = \{x \in \mathbb{R}^S_+ : x \leq x^t\} \cup \{x \in \mathbb{R}^S_+ : p^t \cdot x \leq e\, p^t \cdot x^t\}.$$

Lottery Equivalence and Impatience

A utility function $U : \mathbb{R}^4_+ \to \mathbb{R}$ satisfies lottery equivalence if it agrees with $\trianglerighteq_{\mathrm{LE}}$ defined as follows:

$$((a, b), (a', b')) \trianglerighteq_{\mathrm{LE}} ((a', b'), (a, b))$$

for all (a, b) and (a', b') in $\mathbb{R}^2_+$.

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The utility function $U : \mathbb{R}^4_+ \to \mathbb{R}$ satisfies impatience if

$U((a, b), (a', b')) \geq U((b, a), (a', b'))$ whenever $a \geq b$ and
$U((a, b), (a', b')) \geq U((a, b), (b', a'))$ whenever $a' \geq b'$.

The preorder $\trianglerighteq_{\mathrm{I}}$ corresponding to impatience is

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Obviously, DEU satisfies impatience if $\delta < 1$.

Lottery Equivalence and Impatience

Obviously, DEU satisfies both lottery equivalence and impatience.

We can also test if $\mathcal{O}$ is rationalizable by a utility function that satisfies impatience and lottery equivalence.

Equivalently, such a utility function agrees with

$$\trianglerighteq = \text{tran}(\trianglerighteq_{\text{LE}} \cup \trianglerighteq_{\text{I}}).$$

Table: Pass Rates for DEU properties

	$\bar{e} \geq 0.99$	$\bar{e} \geq 0.95$	$\bar{e} \geq 0.90$
Basic rationality	71.8	90.3	97.1
Lottery Equivalence (LE)	62.1	84.5	93.2
Impatience	65.1	84.5	92.2
LE and Impatience	55.3	79.6	91.3

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Patience	50.5	66.0	73.8

Correlation Neutrality

A utility function $U : \mathbb{R}^4_+ \to \mathbb{R}$ satisfies correlation neutrality if

$$\begin{aligned} U\Big((a,b),(a',b')\Big) &= U\Big((a',b),(a,b')\Big) \text{ and} \\ U\Big((a,b),(a',b')\Big) &= U\Big((a,b'),(a,b)\Big) \end{aligned}$$

for all (a,b) and (a',b') in $\mathbb{R}^2_+$.

DEU satisfies correlation neutrality.

And so does any symmetric time-separable utility function:

$$U((a,b),(a',b')) = H(g(a,a'),g(b,b'))$$

where g is symmetric.

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LE and Impatience	55.3	79.6	91.3
Correlation Neutrality	14.6	22.3	56.3

Correlation Aversion

Let $U : \mathbb{R}^4_+ \to \mathbb{R}$ be a utility function satisfying lottery equivalence. U satisfies correlation aversion if for all payouts $a \geq a'$ and $b \geq b'$

$$U\Big((a', b), (a, b')\Big) \geq U\Big((a, b), (a', b')\Big) \tag{2}$$

If the above inequality is reversed then U is correlation seeking.

Correlation Aversion

Let $U : \mathbb{R}^4_+ \to \mathbb{R}$ be a utility function satisfying lottery equivalence. U satisfies correlation aversion if for all payouts $a \geq a'$ and $b \geq b'$

$$U\Big((a', b), (a, b')\Big) \geq U\Big((a, b), (a', b')\Big) \tag{2}$$

If the above inequality is reversed then U is correlation seeking.

In the Kihlstrom-Mirman form:

$$U(c) = \phi\Big(u(c_{1,1}) + \delta u(c_{1,2})\Big) + \phi\Big(u(c_{2,1}) + \delta u(c_{2,2})\Big)$$

lottery equivalence always holds and impatience holds if $\delta \in (0, 1)$.

Correlation aversion holds if ϕ is concave.

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If the above inequality is reversed then U is correlation seeking.

Table: Pass Rates: Correlation Aversion vs Correlation Seeking (with LE+I)

	$\bar{e} \geq 0.99$	$\bar{e} \geq 0.95$	$\bar{e} \geq 0.90$
Correlation Aversion	51.5	75.7	89.3
Correlation Seeking	16.5	26.2	52.4
Correlation Neutrality	13.6	22.3	50.5

Stochastic Impatience

A utility function $U : \mathbb{R}^4_+ \to \mathbb{R}$ satisfies stochastic impatience (DeJarnette, Dillenberger, Gottlieb, Ortoleva (2020)) if it satisfies lottery equivalence and for all $c \leq b \leq a$,

$$U\Big((a,c),(c,b)\Big) \geq U\Big((b,c),(c,a)\Big).$$

For example, $U((10,0),(0,5)) \geq U((5,0),(0,10))$.

If the inequality is reversed, U obeys stochastic patience.

Stochastic impatience is a consequence of DEU, but not LE + I.

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stochastic impatience holds if u is increasing, $u(r) > 0$ for all $r > 0$ and ϕ has coefficient of relative risk aversion less than 1.

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Table: Pass Rates for DEU properties

	$\bar{e} \geq 0.99$	$\bar{e} \geq 0.95$	$\bar{e} \geq 0.90$
Basic rationality	71.8	90.3	97.1
Lottery Equivalence (LE)	62.1	84.5	93.2
Impatience	65.1	84.5	92.2
LE and Impatience	55.3	79.6	91.3
Stochastic Impatience	53.4	79.6	91.3
Stochastic Patience	42.7	68.0	82.5

Takeaways

There is support for U that satisfies lottery equivalence and impatience, and even stochastic impatience.

There is **little support for correlation neutrality**,

which is enough to guarantee that DEU performs poorly.

There is **strong evidence of correlation aversion**.

Ordinal Dominance

A utility function $U : \mathbb{R}_+^4 \to \mathbb{R}$ satisfies ordinal dominance if U satisfies lottery equivalence and

$$U\Big((a,b),(a,b)\Big) \geq U\Big((a',b'),(a',b')\Big) \implies U\Big((a,b),(a'',b'')\Big) \geq U\Big((a',b'),(a'',b'')\Big)$$

for all $a, a', a'', b, b', b'' \in \mathbb{R}_+$.

Equivalent to U being state separable:

the existence of G and f such that

$$U\Big((a,b),(a',b')\Big) = G\left(f(a,b), f(a',b')\right)$$

We can choose $f(a,b) = U((a,b),(a,b))$, i.e., $f(a,b)$ is the utility when there is no risk.

Ordinal Dominance

Suppose an agent's utility is

$$U\Big((x_{11}, x_{12}), (x_{21}, x_{22})\Big) = G\left(f(x_{11}, x_{12}), f(x_{21}, x_{22})\right).$$

Suppose $\hat{x} = (\hat{x}_1, \hat{x}_2)$ maximizes U in the budget set

$$\{(x_1, x_2) \in \mathbb{R}^4_+ : (\hat{p}_1, \hat{p}_2) \cdot (x_1, x_2) \leq (\hat{p}_1, \hat{p}_2) \cdot (\hat{x}_1, \hat{x}_2)\}.$$

(Notation: $\hat{x}_1 = (\hat{x}_{11}, \hat{x}_{12}) \in \mathbb{R}^2_+,\ \hat{p}_1 = (\hat{p}_{11}, \hat{p}_{12}) \in \mathbb{R}^2_+,$ etc.)

Then $\hat{x}_1$ maximizes $f(x_1)$ in the set

$$\{x_1 = (x_{11}, x_{12}) \in \mathbb{R}^2_+ : \hat{p}_1 \cdot x_1 \leq \hat{p}_1 \cdot \hat{x}_1\}.$$

In other words, if $(\hat{x}_1, \hat{x}_2)$ maximizes overall utility, then $\hat{x}_1$ must maximize the sub-utility in state 1, among consumption streams that cost no more than $\hat{x}_1$.

Similarly, $\hat{x}_2$ maximizes state 2 sub-utility, among all consumption streams in state 2 that cost no more than $\hat{x}_2$.

Ordinal Dominance

Let $\mathcal{O}_{\text{split}} = \{(x_1^1, p_1^1), \ldots, (x_1^N, p_1^N)\} \cup \{(x_2^1, p_2^1), \ldots, (x_2^N, p_2^N)\}$

A necessary (but not sufficient) condition for $\mathcal{O}$ to be rationalized by U that satisfies impatience and weak separability is that

$\mathcal{O}_{\text{split}}$ can be rationalized by some strictly increasing continuous utility function $f : \mathbb{R}_+^2 \to \mathbb{R}$ such that $f(a, b) \geq f(b, a)$ if $a \geq b$.

Table: Ordinal Dominance Test using $\mathcal{O}_{\text{split}}$

	$\bar{e} \geq 0.99$	$\bar{e} \geq 0.95$	$\bar{e} \geq 0.90$
Impatient subutility	65.0	78.6	84.5

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The paper goes on to estimate a parametric version of the Kihlstrom-Mirman model for each subject:

$$U(c) = \phi\Big(u(c_{1,1}) + \delta u(c_{1,2})\Big) + \phi\Big(u(c_{2,1}) + \delta u(c_{2,2})\Big)$$

Conclusion

New Super GARP revealed preference techniques allow us to test for structural properties on utility functions.

Among the features of the DEU model, there is support for

lottery equivalence, impatience, correlation aversion,

stochastic impatience, and state-separability.

But not for correlation neutrality.