Nonparametric Analysis of Monotone Choice

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Introduction

Empirical investigation of consumer demand often uses a nonparametric approach

Which relies on *qualitative* features of demand derived from revealed preference.

This approach can also be useful for studying games.

We focus on the class of complete information games with strategic complementarity.

Example: IO entry model of airlines

Berry (1992), Ciliberto and Tamer (2009), Kline and Tamer (2016).

Assumptions:

The econometrician observes many markets, defined as trips between two airports.

In each market, there are two airline firms: $i \in \{1, 2\}$

For each market, we observe the covariates (x_1, x_2) and the entry decisions of firms: (N, N), (N, E), (E, N), or (E, E).

Information can be summarized by distribution of firm decisions at realized covariates (x_1, x_2).

		Firm 2	
		N	E
Firm 1	N	$P(N,N \mid x_1, x_2)$	$P(N,E \mid x_1, x_2)$
	E	$P(E,N \mid x_1, x_2)$	$P(E,E \mid x_1, x_2)$

Example: IO entry model of airlines

In Kline and Tamer (2016), covariates are the following:

Market Presence (which is firm-specific) P_1, $P_2 \in \{0, 1\}$ and

Market Size $S \in \{0, 1\}$

Thus all markets can be partitioned into precisely eight types: $(1, 1, 1)$, $(0, 1, 1)$, $(0, 0, 1)$, etc.

In each market, we observe the the entry decision of the pair of firms: (N, N), (N, E), (E, N), or (E, E).

Thus the data consists of precisely eight tables.

		Firm 2	
		N	E
Firm 1	N	P(N,N $\mid x_1, x_2$)	P(N,E $\mid x_1, x_2$)
	E	P(E,N $\mid x_1, x_2$)	P(E,E $\mid x_1, x_2$)

Example: IO entry model of airlines

Distribution of entry choices (from Kline and Tamer (2016))

Covariates $= (0,0,0)$				Covariates $= (0,1,0)$			
N,N	N,E	E,N	E,E	N,N	N,E	E,N	E,E
30.37	68.21	0.55	**0.87**	19	78.51	0.26	2.23
Covariates $= (1,0,0)$				Covariates $= (1,1,0)$			
N,N	N,E	E,N	E,E	N,N	N,E	E,N	E,E
19.38	36.71	25.33	18.58	12.15	54.22	4.99	28.64
Covariates $= (0,0,1)$				Covariates $= (0,1,1)$			
N,N	N,E	E,N	E,E	N,N	N,E	E,N	E,E
15.88	82.28	0.12	1.73	7.80	88.93	0	3.27
Covariates $= (1,0,1)$				Covariates $= (1,1,1)$			
N,N	N,E	E,N	E,E	N,N	N,E	E,N	E,E
10.64	32.64	30.58	26.14	5.53	50.07	2.14	**42.26**

Standard analysis of this data

This involves a parametric specification of payoff functions.

$$\Pi_1(y_1, y_2, x_1, \varepsilon_1) = \begin{cases} \alpha_1' x_1 + \delta_1 1(y_2 = \mathrm{E}) + \varepsilon_1 & \text{if} \quad y_1 = \mathrm{E} \\ 0 & \text{if} \quad y_1 = \mathrm{N} \end{cases}$$

$$\Pi_2(y_1, y_2, x_2, \varepsilon_2) = \begin{cases} \alpha_2' x_2 + \delta_2 1(y_1 = \mathrm{E}) + \varepsilon_2 & \text{if} \quad y_2 = \mathrm{E} \\ 0 & \text{if} \quad y_2 = \mathrm{N} \end{cases}$$

Estimate α_1', δ_1 and α_2', δ_2

Common assumptions:

- interaction effects, δ_1 and δ_2, are negative
- distribution of $(\varepsilon_1, \varepsilon_2)$ belongs to a known family
- $(\varepsilon_1, \varepsilon_2)$ is independent of (x_1, x_2)
- firms play pure strategy Nash equilibrium

Our analysis

We provide a nonparametric test of the joint hypothesis that firms are

(i) playing pure strategy Nash equilibria

(ii) in games of strategic substitutes.

(Note: pure strategy Nash equilibrium always exists in two-player games of substitutes.)

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Features of our approach

- Nonparametric
- No assumption on equilibrium selection mechanism
- Very general approach to model heterogeneity
  - no assumption on the joint distribution of $(\varepsilon_1, \varepsilon_2)$
  - no assumption on group formation

But we do assume $(\varepsilon_1, \varepsilon_2)$ is independent of (x_1, x_2).
In other words, we require payoff functions to be distributed independently of the covariates.

How does the idea work?

Key observation: we should focus on behavior and not payoffs.

A realization of $(\varepsilon_1, \varepsilon_2)$ leads to $\Pi_1(y_1, y_2, x_1, \varepsilon_1)$ and $\Pi_2(y_1, y_2, x_2, \varepsilon_2)$.

Combined with an equilibrium selection rule, these payoffs induce a sequence of Nash equilibrium choices across different covariates (x_1, x_2).

This map from covariates to action profiles correspond to a group type.

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This map from covariates to action profiles correspond to a group type.

Data can only convey information up to group types — *nothing finer*:

distinct values of $(\varepsilon_1, \varepsilon_2)$ are indistinguishable if they generate the same group type.

How does the idea work?

Suppose that firm i's payoff is
(i) increasing with x_i and (ii) falls with entry of the other firm.

This will *restrict* the set of group types. Only certain types are consistent with these sign restrictions.

Assume x_1 is fixed and x_2 takes three values: $(0,0)$, $(0,1)$ and $(1,0)$.

(E,E) at $(0,0)$, (E,N) at $(0,1)$, and (N,N) at $(1,0)$ is impossible.

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(E,N) at $(0,0)$, (N,E) at $(0,1)$ and (E,E) at $(1,0)$ is impossible.

As possible joint choices and covariate values are finite, we can enumerate ALL consistent group types.

How does the idea work?

We observe the distribution across joint actions at $x_2 = (0,0)$, $(0,1)$, and $(1,0)$. (x_1 held fixed.)

		Firm 2	
		N	E
Firm 1	N	$P(N,N \mid x_1, x_2)$	$P(N,E \mid x_1, x_2)$
	E	$P(E,N \mid x_1, x_2)$	$P(E,E \mid x_1, x_2)$

We want to test the hypothesis that firms

(i) are playing pure strategy Nash equilibria;
(ii) have payoff functions with given sign restrictions.

First, we find all group types consistent with (i) and (ii).

The data is consistent with our hypothesis if we can find a distribution over consistent group types that explain the observations.

How does the idea work?

Suppose we have the following data (for some fixed x_1)

$x_2 = (0,0)$		Firm 2	
		N	E
Firm 1	N	3/12	3/12
	E	4/12	2/12

$x_2 = (0,1)$		Firm 2	
		N	E
Firm 1	N	1/12	5/12
	E	3/12	3/12

$x_2 = (1,0)$		Firm 2	
		N	E
Firm 1	N	2/12	4/12
	E	2/12	4/12

How does the idea work?

Data can be rationalized by the following group types.

Type	Weight	$x_2 = (0,0)$				$x_2 = (0,1)$				$x_2 = (1,0)$			
		Action profiles				Action profiles				Action profiles			
		N,N	N,E	E,N	E,E	N,N	N,E	E,N	E,E	N,N	N,E	E,N	E,E
1				●					●			●	
2		●					●			●			
3				●				●					●
4				●				●				●	
5		●				●					●		
6					●				●				●
7			●				●				●		

How does the idea work?

Data can be rationalized as follows.

Type	Weight	$x_2 = (0,0)$				$x_2 = (0,1)$				$x_2 = (1,0)$			
		Action profiles				Action profiles				Action profiles			
		N,N	N,E	E,N	E,E	N,N	N,E	E,N	E,E	N,N	N,E	E,N	E,E
1	1/12			1/12					1/12			1/12	
2	2/12	2/12					2/12			2/12			
3	2/12			2/12				2/12					2/12
4	1/12			1/12				1/12				1/12	
5	1/12	1/12				1/12					1/12		
6	2/12				2/12				2/12				2/12
7	3/12		3/12				3/12				3/12		
Sum	1	3/12	3/12	4/12	2/12	1/12	5/12	3/12	3/12	2/12	4/12	2/12	4/12

$x_2 = (0,0)$		Firm 2	
		N	E
Firm 1	N	3/12	3/12
	E	4/12	2/12

$x_2 = (0,1)$		Firm 2	
		N	E
Firm 1	N	1/12	5/12
	E	3/12	3/12

$x_2 = (1,0)$		Firm 2	
		N	E
Firm 1	N	2/12	4/12
	E	2/12	4/12

Our general approach

Step 1

Determine ALL the group types that are consistent with the sign restrictions on payoff functions.

The paper formulates a Revealed Monotonicity Axiom:

can be used to check for consistency of a group type with pure strategy Nash equilibrium play in games of strategic complements.

If a group type obeys the axiom, then it is possible to construct payoff functions that

(i) obey single crossing conditions and

(ii) have observed actions as optimal.

Our general approach

Step 1

Determine ALL the group types that are consistent with the sign restrictions on payoff functions.

The Revealed Monotonicity Axiom provides a procedure to do this.

Step 2

Determine whether or not there is a distribution over the consistent types identified in Step 1 that explains the data.

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Determine ALL the group types that are consistent with the sign restrictions on payoff functions.

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Determine whether or not there is a distribution over the consistent types identified in Step 1 that explains the data.

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If data set passes the test, we can perform further analyses. For example

(i) estimate the fraction of types of with specific features or

(ii) make estimates of the distribution of equilibrium behavior at out-of-sample covariate values.

Games with single-crossing payoff functions

A group consists of $\mathcal{N} = \{1, 2, ..., n\}$ agents.

Agent $i \in \mathcal{N}$ chooses an action y_i from an action space Y_i.
We assume is finite and totally ordered.

The joint action profile of the group is $\mathbf{y} \in \mathbf{Y} = \times_{i \in \mathcal{N}} Y_i$.

For each $i \in \mathcal{N}$, there is an $M(i)$-dimensional covariate $x_i \in X_i \subset \mathbb{R}^{M(i)}$.

Denote the profile of covariates by $\mathbf{x} = (x_i : i \in \mathcal{N})$.

The payoff of agent i is given by $\Pi_i : Y_i \times \mathbf{Y}_{-i} \times X_i \to \mathbb{R}$.

$\mathbf{\Pi} = (\Pi_i : i \in \mathcal{N})$ denotes a profile of payoff functions.

Games with single-crossing payoff functions

$(\mathbf{\Pi}, \mathbf{x})$ induces a game of complete information $\mathrm{G}(\mathbf{\Pi}, \mathbf{x})$.

Let $\mathbf{X} \subset \times_{i \in \mathcal{N}} X_i$ denote the set of conceivable joint realizations of covariates $\mathbf{x}$.

We denote the best response of each player i at $(\mathbf{y}_{-i}, x_i)$ by

$$\mathrm{BR}_i(\mathbf{y}_{-i}, x_i) = \mathrm{argmax}_{y_i \in Y_i} \Pi_i \left(y_i, \mathbf{y}_{-i}, x_i\right).$$

We assume agents have *strict* preferences over actions, so $\mathrm{BR}_i(\mathbf{y}_{-i}, x_i)$ is unique.

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The set of pure strategy Nash equilibria (PSNE) of this game is

$$\mathrm{NE}(\boldsymbol{\Pi}, \mathbf{x}) = \{\mathbf{y}^* \in \mathbf{Y} : y_i^* = \mathrm{BR}_i(\mathbf{y}^*_{-i}, x_i) \text{ for all } i \in \mathcal{N}\}.$$

We are interested in games where payoff functions obey single-crossing conditions (Milgrom and Shannon, 1994).

Games with single-crossing payoff functions

Definition.

The payoff function Π_i has single-crossing differences in $(y_i; (\mathbf{y}_{-i}, x_i))$ if the following condition holds:

for every $y_i'' > y_i'$ and $(\mathbf{y}''_{-i}, x_i'') > (\mathbf{y}'_{-i}, x_i')$ in the product order,

$$\Pi_i\left(y_i'', \mathbf{y}'_{-i}, x_i'\right) > \Pi_i\left(y_i', \mathbf{y}'_{-i}, x_i'\right) \Longrightarrow \Pi_i\left(y_i'', \mathbf{y}''_{-i}, x_i''\right) > \Pi_i\left(y_i', \mathbf{y}''_{-i}, x_i''\right).$$

We refer to Π_i as a single-crossing payoff function.

We write $\mathbf{\Pi} = (\Pi_i)_{i \in \mathcal{N}} \in \mathcal{SC}$ if Π_i is a single-crossing function for all i.

Games with single-crossing payoff functions

Background Result.

If $\mathbf{\Pi} \in \mathcal{SC}$, the family of games $\{\mathrm{G}(\mathbf{\Pi}, \mathbf{x}) : \mathbf{x} \in \mathbf{X}\}$ has the following properties:

(i) $\mathrm{BR}_i(\mathbf{y}_{-i}, x_i)$ is increasing in $(\mathbf{y}_{-i}, x_i)$ for each $i \in \mathcal{N}$ and
(ii) $\mathrm{NE}(\mathbf{\Pi}, \mathbf{x})$ is non-empty.

The main theoretical result in the paper is a revealed preference result: the converse of this statement.

Games with single-crossing payoff functions

A group type is a function $\mathrm{B}\colon \hat{\mathbf{X}} \to \mathbf{Y}$ that associates a profile of actions $\mathbf{y}$ to each covariate $\mathbf{x} \in \hat{\mathbf{X}}$.

Definition. A group type $\mathrm{B} : \hat{\mathbf{X}} \to \mathbf{Y}$ is a single-crossing group type if there exists a profile of payoff functions $\mathbf{\Pi}$ in $\mathcal{SC}$ such that $\mathrm{B}(\mathbf{x}) \subset \mathrm{NE}(\mathbf{\Pi}, \mathbf{x})$ for all $\mathbf{x} \in \hat{\mathbf{X}}$.

Definition. A group type $\mathrm{B} : \hat{\mathbf{X}} \to \mathbf{Y}$ obeys the revealed monotonicity (RM) axiom, if for each $\mathbf{x}', \mathbf{x}'' \in \hat{\mathbf{X}}$, $\mathbf{y}' = \mathrm{B}(\mathbf{x}')$, and $\mathbf{y}'' = \mathrm{B}(\mathbf{x}'')$,

$$(\mathbf{y}''_{-i}, x''_i) \geqslant (\mathbf{y}'_{-i}, x'_i) \implies y''_i \geqslant y'_i \text{ for each } i \in \mathcal{N}.$$

Games with single-crossing payoff functions

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$$(\mathbf{y}''_{-i}, x''_i) \geqslant (\mathbf{y}'_{-i}, x'_i) \Longrightarrow y''_i \geqslant y'_i \text{ for each } i \in \mathcal{N}.$$

Main Theorem 1. $\mathrm{B} : \hat{\mathbf{X}} \to \mathbf{Y}$ is a single-crossing group type if and only if it satisfies the RM axiom.

$\mathcal{SC}$-rationalizable distributions

There is a population of groups.

For each group, the set of agents is $\mathcal{N} = \{1, 2, ..., n\}$.

Every group plays the (same) game of complete information $\mathsf{G}(\boldsymbol{\Pi}, \mathbf{x})$.

Groups have heterogenous preferences: at a given covariate value $\mathbf{x}$, different groups will take different joint actions.

This generates a conditional distribution over joint actions $\mathbf{y} \in \mathbf{Y}$.

We denote this by $\mathsf{P}(\cdot \mid \mathbf{x})$.

Assume that $\mathsf{P}(\cdot \mid \mathbf{x})$ is known for $\mathbf{x}$ in $\hat{\mathbf{X}}$, a *finite* subset of $\mathbf{X}$.

The set of choice distributions

$$\mathcal{P} = \left\{\mathsf{P}(\cdot \mid \mathbf{x}) : \mathbf{x} \in \hat{\mathbf{X}}\right\}$$

constitutes an idealized dataset.

$\mathcal{SC}$-rationalizable distributions

When is the idealized dataset $\mathcal{P} = \left\{ \mathsf{P}(\cdot \mid \mathbf{x}) : \mathbf{x} \in \hat{\mathbf{X}} \right\}$ consistent with PSNE play in games with single-crossing payoff functions?

Different choices across groups can arise from heterogeneity in payoff functions and heterogeneity in equilibrium selection.

$\mathcal{SC}$-rationalizable distributions

When is the idealized dataset $\mathcal{P} = \left\{\mathsf{P}(\cdot \mid \mathbf{x}) : \mathbf{x} \in \hat{\mathbf{X}}\right\}$ consistent with PSNE play in games with single-crossing payoff functions?

Different choices across groups can arise from heterogeneity in payoff functions and heterogeneity in equilibrium selection.

We denote an equilibrium selection rule by $\lambda\left(\mathbf{y}|\mathbf{\Pi}, \mathbf{x}\right)$.
(Therefore, $\lambda\left(\mathbf{y}|\mathbf{\Pi}, \mathbf{x}\right) = 0$ for all $\mathbf{y} \notin \mathsf{NE}(\mathbf{\Pi}, \mathbf{x})$ and $\sum_{\mathbf{y} \in \mathbf{Y}} \lambda(\mathbf{y}|\mathbf{\Pi}, \mathbf{x}) = 1$.)

Definition. A distribution $\mathrm{P}_{\mathbf{\Pi}}$ on $\mathbf{\Pi} = (\Pi_i)_{i \in \mathcal{N}}$ rationalizes $\mathcal{P}$ if there is an equilibrium selection rule $\lambda\left(\cdot|\mathbf{\Pi}, \mathbf{x}\right)$ such that

$$\mathrm{P}(\mathbf{y}|\mathbf{x}) = \int \lambda\left(\mathbf{y}|\mathbf{\Pi}, \mathbf{x}\right) d\mathrm{P}_{\mathbf{\Pi}} \textit{ for all } \mathbf{y} \in \mathbf{Y} \textit{ and all } \mathbf{x} \in \hat{\mathbf{X}}.$$

If $\mathrm{P}_{\mathbf{\Pi}}$ has support on $\mathcal{SC}$, then we say that $\mathcal{P}$ is $\mathcal{SC}$-rationalizable.

$\mathcal{SC}$-rationalizable distributions

Main Theorem 2.

Let $\mathcal{B}$ be the set of $\mathcal{SC}$-rationalizable group types.

$\mathcal{P} = \left\{\mathrm{P}(\cdot \mid \mathbf{x}) : \mathbf{x} \in \hat{\mathbf{X}}\right\}$ is $\mathcal{SC}$-rationalizable if and only if there exists a distribution $\tau = (\tau^B)_{B \in \mathcal{B}}$ on $\mathcal{B}$ such that the following holds:

$$\mathrm{P}(\mathbf{y} \mid \mathbf{x}) = \sum_{\{\mathrm{B} \in \mathcal{B}:\ \mathrm{B}(\mathbf{x}) = \mathbf{y}\}} \tau^{\mathrm{B}} \text{ for all } \mathbf{y} \in \mathbf{Y} \text{ and } \mathbf{x} \in \hat{\mathbf{X}}.$$

Inference

We can extract information on the distributions $\mathrm{P}_{\mathbf{\Pi}}$ that rationalize $\mathcal{P}$.

Suppose we are interested in the incidence of a given subset of single-crossing payoff function profiles $\mathcal{SC}^*$.

Let $\mathcal{B}^* = \left\{\mathrm{B} \in \mathcal{B} : \text{there is } \mathbf{\Pi} \in \mathcal{SC}^* \text{ that rationalizes } \mathrm{B}\right\}$.

Corollary. Suppose $\mathcal{P} = \left\{\mathsf{P}(\cdot \mid \mathbf{x}) : \mathbf{x} \in \hat{\mathbf{X}}\right\}$ is $\mathcal{SC}$-rationalizable and so there is $\tau = (\tau^B)_{B \in \mathcal{B}}$ on $\mathcal{B}$ such that

$$\mathrm{P}(\mathbf{y} \mid \mathbf{x}) = \sum\nolimits_{\{\mathrm{B} \in \mathcal{B}:\ \mathsf{B}(\mathbf{x}) = \mathbf{y}\}} \tau^{\mathrm{B}} \text{ for all } \mathbf{y} \in \mathbf{Y} \text{ and } \mathbf{x} \in \hat{\mathbf{X}}. \tag{1}$$

Then

$$\max\left\{\int_{\mathbf{\Pi} \in \mathcal{SC}^*} d\mathrm{P}_{\mathbf{\Pi}} : \mathrm{P}_{\mathbf{\Pi}} \text{ rationalizes } \mathcal{P}\right\} = \max\left\{\sum\nolimits_{\mathrm{B} \in \mathcal{B}^*} \tau^{\mathrm{B}} : (\tau^{\mathrm{B}})_{\mathrm{B} \in \mathcal{B}} \text{ solves } (1)\right\}.$$

Distribution of entry choices in the data

Data set is from Kline and Tamer (2016). Covariates are the following:

Market Presence (which is firm-specific) $P_1,\ P_2 \in \{0,1\}$ and

Market Size $S \in \{0,1\}$

Thus all markets can be partitioned into precisely eight types.

Covariates = $(0,0,0)$				Covariates = $(0,1,0)$			
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30.37	68.21	0.55	**0.87**	19	78.51	0.26	2.23
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15.88	82.28	0.12	1.73	7.80	88.93	0	3.27
Covariates = $(1,0,1)$				Covariates = $(1,1,1)$			
N,N	N,E	E,N	E,E	N,N	N,E	E,N	E,E
10.64	32.64	30.58	26.14	5.53	50.07	2.14	**42.26**

Hypothesis Testing

Hypothesis:

This data set is generated by pure strategy Nash equilibria where firm i's payoff has the following properties:

(i) it is increasing in its market presence P_i and in market size S;
(ii) its payoff when it enters is strictly higher if the other firm stays out.

There are four possible joint choices for each of the eight covariate values.

Thus there are $4^8 = 65,536$ group types.

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There are four possible joint choices for each of the eight covariate values.

Thus there are $4^8 = 65,536$ group types.

Of these, 482 are single-crossing group types.

	$(0,0,0)$	$(0,1,0)$	$(1,0,0)$	...	$(1,1,1)$	
type 1	N,N	E,E	E,E	...	N,N	RM axiom ✓
type 2	E,E	E,E	E,E	...	N,E	RM axiom ×
...	...	...		...	...	...

Hypothesis Testing

Testing the hypothesis requires checking whether a system of linear equations

$$A\tau = p$$
$$A \text{ is } 32 \times 482,\ x \text{ is } 482 \times 1, \text{ and } b \text{ is } 32 \times 1$$

has a *positive* solution in τ.

- A is a matrix of 0's and 1's that captures all consistent types
  - each column captures the behavior of a type under all covariate values
- p is the observed distribution of entry choices
- τ is a possible distribution of behavioral types.

Hypothesis Testing

- Data is not consistent with our hypothesis
- At least one violation is

$$P(N,N|1,1,0) + P(E,N|1,1,0) = 17.14\% < 19\% = P(N,N|0,1,0)$$

Hypothesis Testing

Closest compatible choice distribution is quite similar!

Covariates $= (0,0,0)$				Covariates $= (0,1,0)$			
N,N	N,E	E,N	E,E	N,N	N,E	E,N	E,E
30.06	67.90	0.86	1.18	18.29	78.96	0.71	2.05
Covariates $= (1,0,0)$				Covariates $= (1,1,0)$			
N,N	N,E	E,N	E,E	N,N	N,E	E,N	E,E
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Covariates $= (0,0,1)$				Covariates $= (0,1,1)$			
N,N	N,E	E,N	E,E	N,N	N,E	E,N	E,E
15.46	81.86	0.54	2.15	7.86	89.19	0.26	2.69
Covariates $= (1,0,1)$				Covariates $= (1,1,1)$			
N,N	N,E	E,N	E,E	N,N	N,E	E,N	E,E
10.64	32.64	30.58	26.14	5.63	49.98	2.24	42.16

Hypothesis Testing

Kitamura and Stoye (2017)

The Null-Hypothesis is

$$\text{(H)}\quad \min_{x\in\mathbb{R}_+^{482}} (b - Ax)' (b - Ax) = 0.$$

The sample counterpart is

$$J_N = N \min_{x\in\mathbb{R}_+^{482}} \left(\hat{b} - Ax\right)' \left(\hat{b} - Ax\right) = 0.$$

p-value is about 15%.

Thus, we don't reject the Null-Hypothesis.

Application: estimating non-strategic types

Out of the 482 consistent types, 36 are also consistent with non-strategic interactions, i.e.,

the entry decision of both airlines can depend on covariates but not on the entry decision of the other airline

We can estimate bounds for the set of 36 non-strategic types

Upper bound is 75%

Conclusion: strategic interaction is crucial to explaining the data.

Application: bounds on NE

Given a strategy profile $\overline{\mathbf{y}}$ and covariate $\overline{\mathbf{x}}$, we pose the following question:

among all $\mathcal{SC}$-rationalizations of $\mathcal{P}$, what is the greatest fraction of groups which could have $\overline{\mathbf{y}}$ as PSNE at $\overline{\mathbf{x}}$?

This is potentially bigger than $\mathrm{P}(\overline{\mathbf{y}} \mid \overline{\mathbf{x}})$, the *observed* fraction of groups in the population that play $\overline{\mathbf{y}}$ at $\overline{\mathbf{x}}$:

In our empirical application, if (E, E) or (N, N) is played by a pair of firms, then it has to be their unique equilibrium, but any pair that plays (E, N) may also have (N, E) as another (albeit unselected) equilibrium.

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The distinction is relevant: if the gap is small, then we are sure that changing the equilibrium selection scheme *cannot* significantly increase the frequency with which $\overline{\mathbf{y}}$ is played.

An earlier analysis of this question is in Aradillas-Lopez (2011).

Application: bounds on NE

$(0,0,0)$		$(0,1,0)$		$(1,0,0)$		$(1,1,0)$	
(N, E)	(E, N)	(N, E)	(E, N)	(N, E)	(E, N)	(N, E)	(E, N)
0.699	0.544	0.815	0.503	0.503	0.644	0.558	0.555
0.682	0.006	0.785	0.003	0.367	0.253	0.542	0.050
$(0,0,1)$		$(0,1,1)$		$(1,0,1)$		$(1,1,1)$	
(N, E)	(E, N)	(N, E)	(E, N)	(N, E)	(E, N)	(N, E)	(E, N)
0.841	0.616	0.913	0.496	0.485	0.661	0.523	0.497
0.832	0.001	0.910	0.000	0.326	0.306	0.501	0.021

Table: Probability bounds for equilibrium action profiles

Conclusion

- We provide a nonparametric test of strategic complementarity in games.
- The procedure can be developed to recover properties of the model and make out-of-sample predictions of equilibrium behavior.
- The procedure is computationally feasible, especially when combined with column generation.